

How to get protons in the LHC? (or electrons in SuperB and the ILC)

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Lectures overview

I. How to produce protons for the LHC?

II. How to get protons in the LHC?

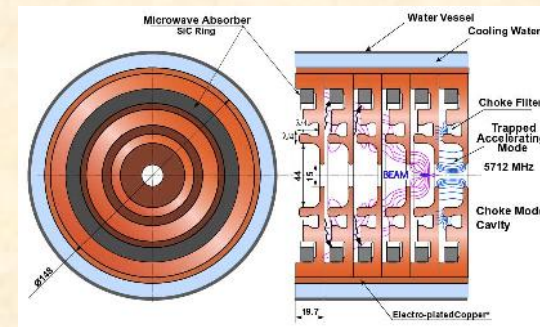
- Magnets
- Beam dynamics
- Beam-beam effects

III. How to “see” protons in the LHC?

*IV. What can you do with a particle accelerator
(apart from hunting the Higgs)?*

Where are we?

- Yesterday we have seen how to produce protons, ions and electrons using different type of sources.
- We have also seen how to accelerate them.
- In the LHC acceleration is done using superconducting cavities.
- But there is still long way before the protons enter the LHC.
- We must also understand what are the challenges to keep them in the LHC, especially when there are collisions.



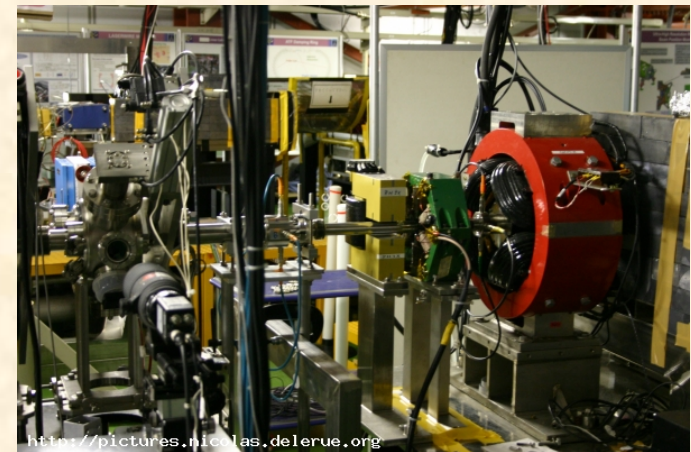
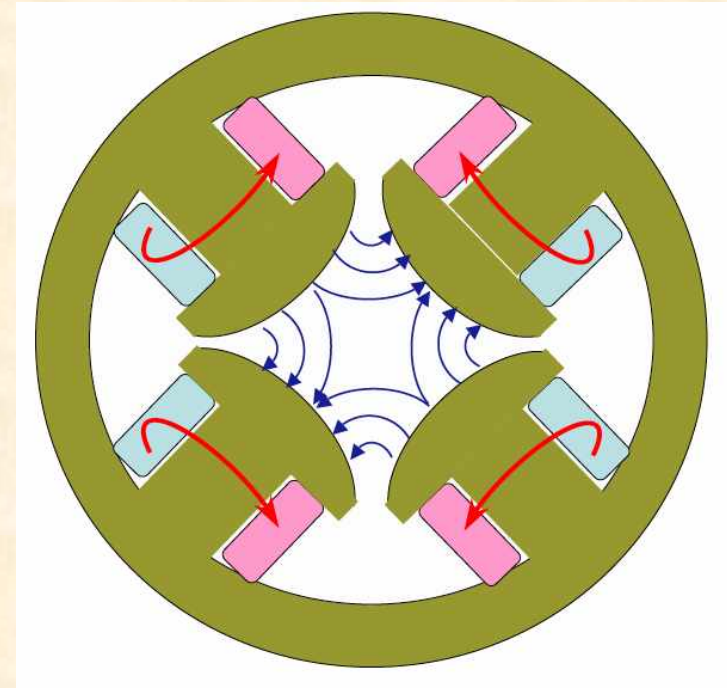
How to control where the particles go?

- Electric and magnetic fields can deflect charged particles.
- In an electric field the particles get accelerated.
- In magnetic field the direction of the particles is changed but not their energy.
=> it is preferable to use magnetic field (usually electromagnets) to control a beam.
- Magnets are also more efficient.



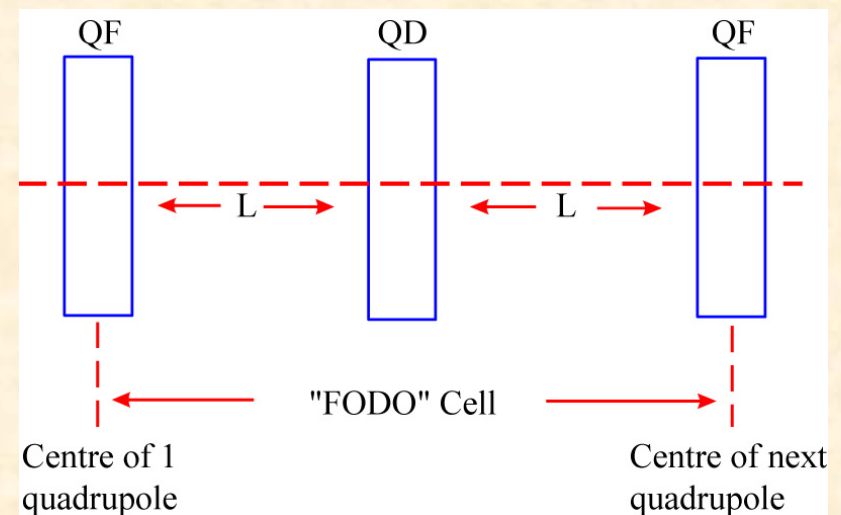
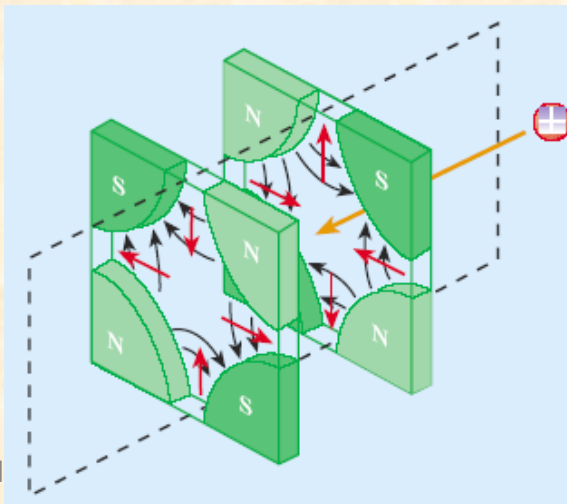
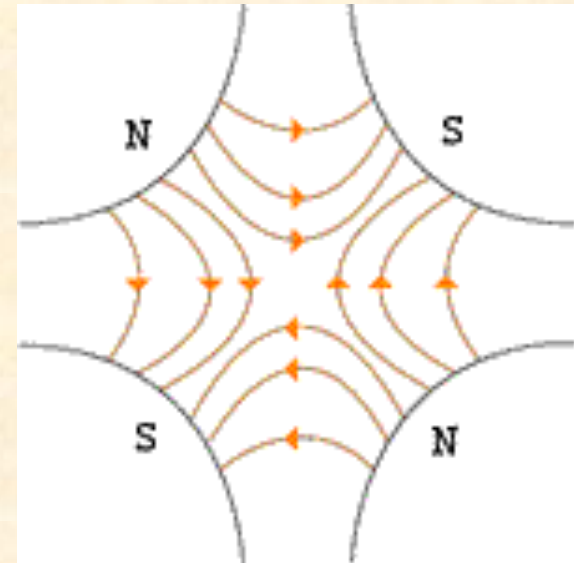
Beam focussing

- A regular magnet (dipole) will create a field that will bend the beam in one direction.
- To change the size of the beam a different type of magnets called quadrupoles need to be used.
- Quadrupoles create intense fields for off-axis particles but do not disturb particles on the axis.



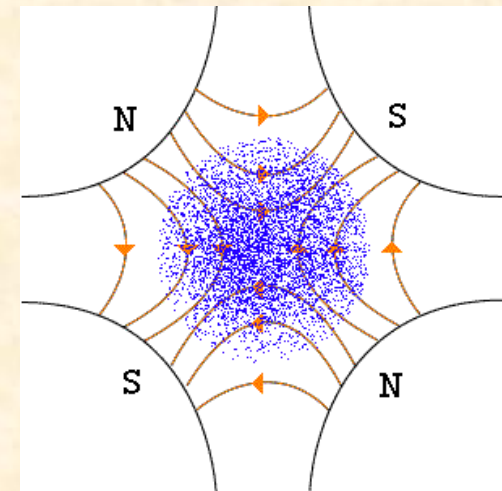
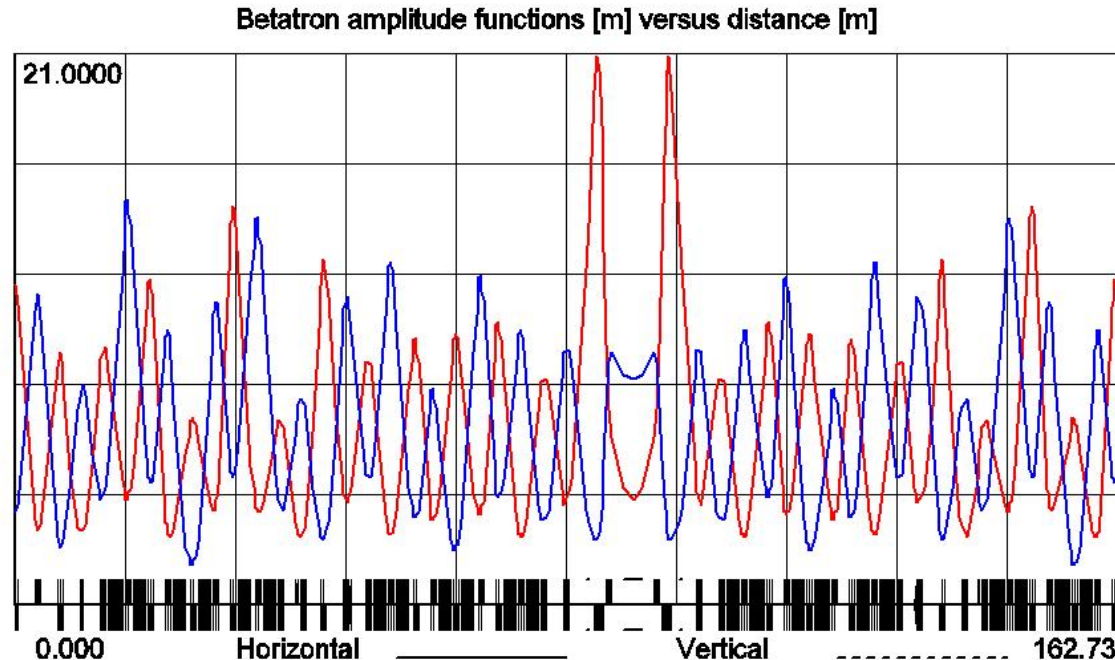
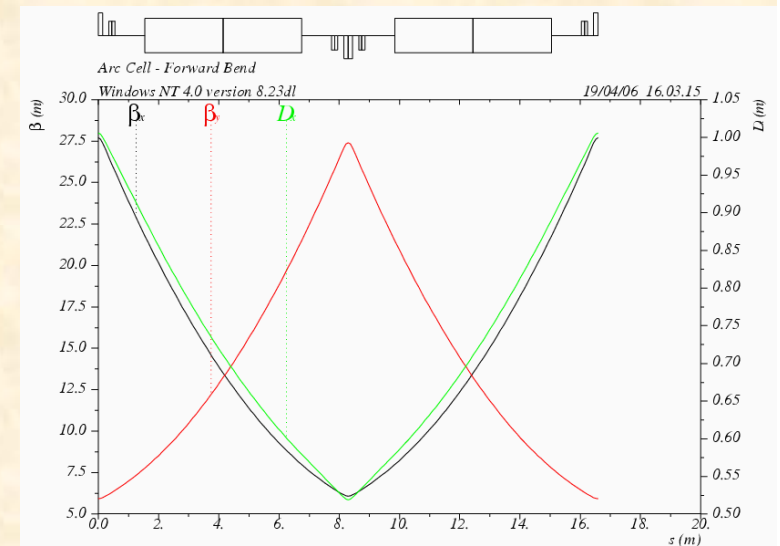
FODO cell

- A quadrupole will focus the beam in one plane but defocus it in the other plane.
- To have a net focussing effect, 2 quadrupoles are used, one focussing in one plane and the other one focussing in the other plane.



Accelerator lattice

- In an accelerator there is a large number of quadrupoles to keep the beam size under control.
- This is called the lattice of the accelerator.
- The magnet strength are characterised by a “beta” function.



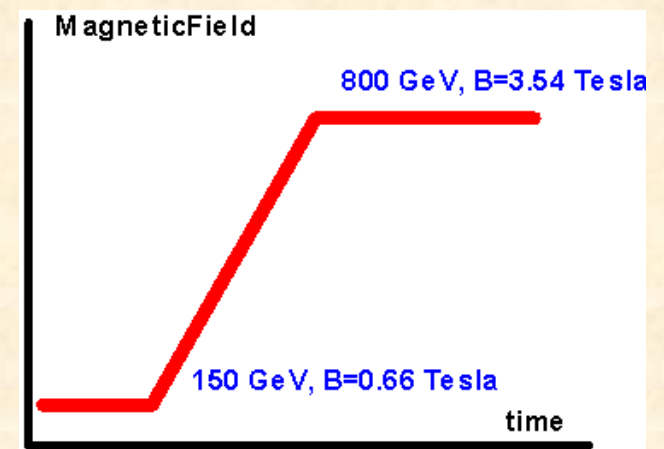
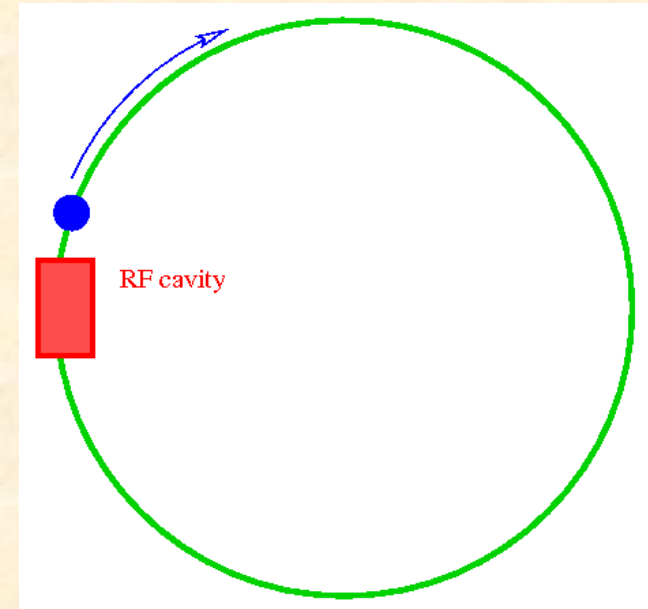
Synchrotrons

Dipole magnets are then used to make the particles follow a circular orbit.

This can be used instead of having a large number of cavities to accelerate the particles, by using a single cavity in which the particles pass several times.

As the particles gain energy the radius of curvature of their orbit in a constant field increases

=> The field of the dipoles has to be increased as the particles gain energy to keep a constant orbit.



Magnetic rigidity

- The bending power of a magnet depends on the beam energy.

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

- It is often useful to use the “magnetic rigidity” of a beam.

$$B\rho = \frac{p}{q}$$

- One should note that rings often have straight section. So the magnetic radius of an accelerator is smaller than its geometrical radius.

$$B[T]\rho[m] = 3.34p[GeV/c]$$

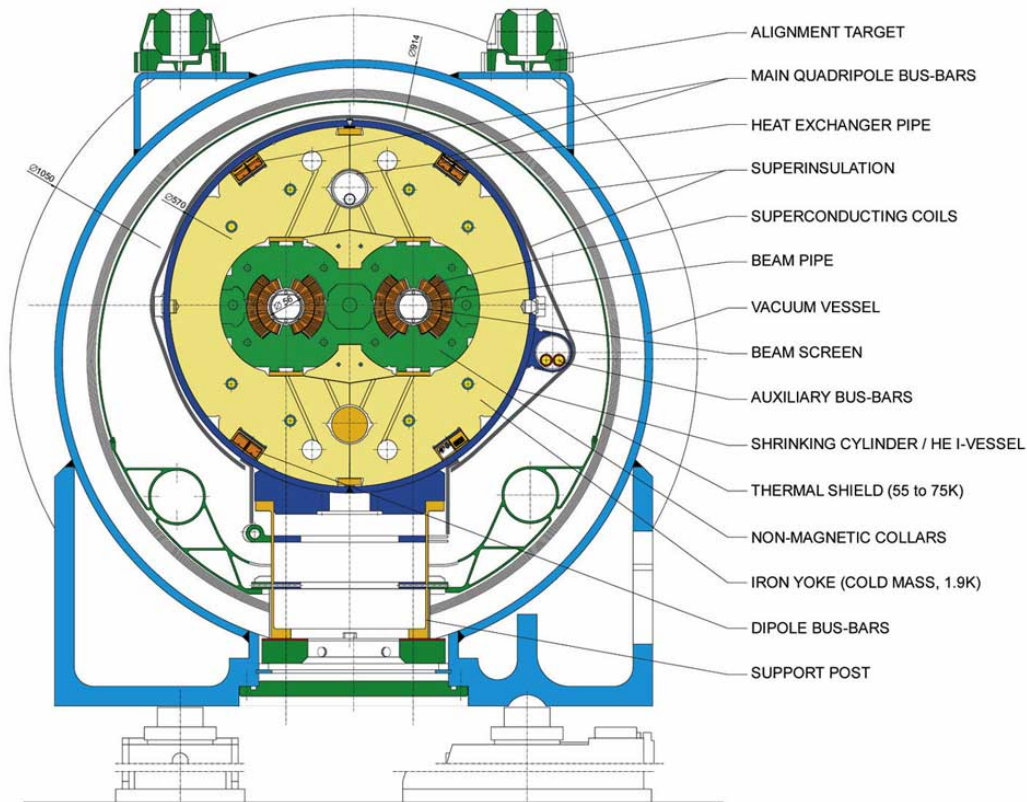
$$B[T] = \frac{3.34 * 4000[GeV/c]}{2804[m]} = 4.8[T]$$

- The LHC has a magnetic radius of 2804m.

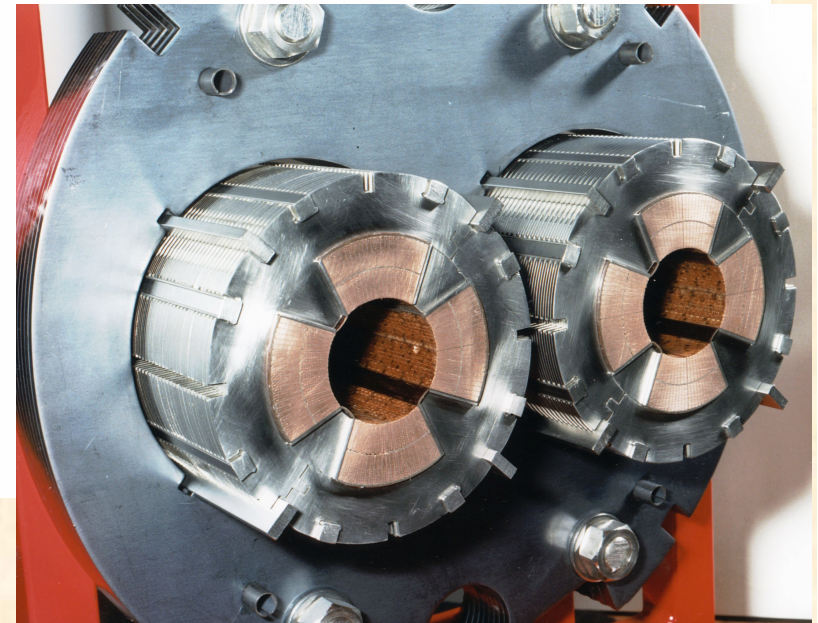
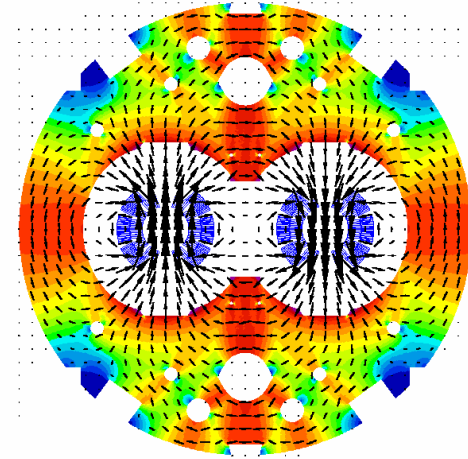
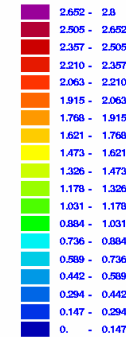
LHC Magnets

LHC DIPOLE : STANDARD CROSS-SECTION

CERN AC/DI/MM - HE107 - 30 04 1999

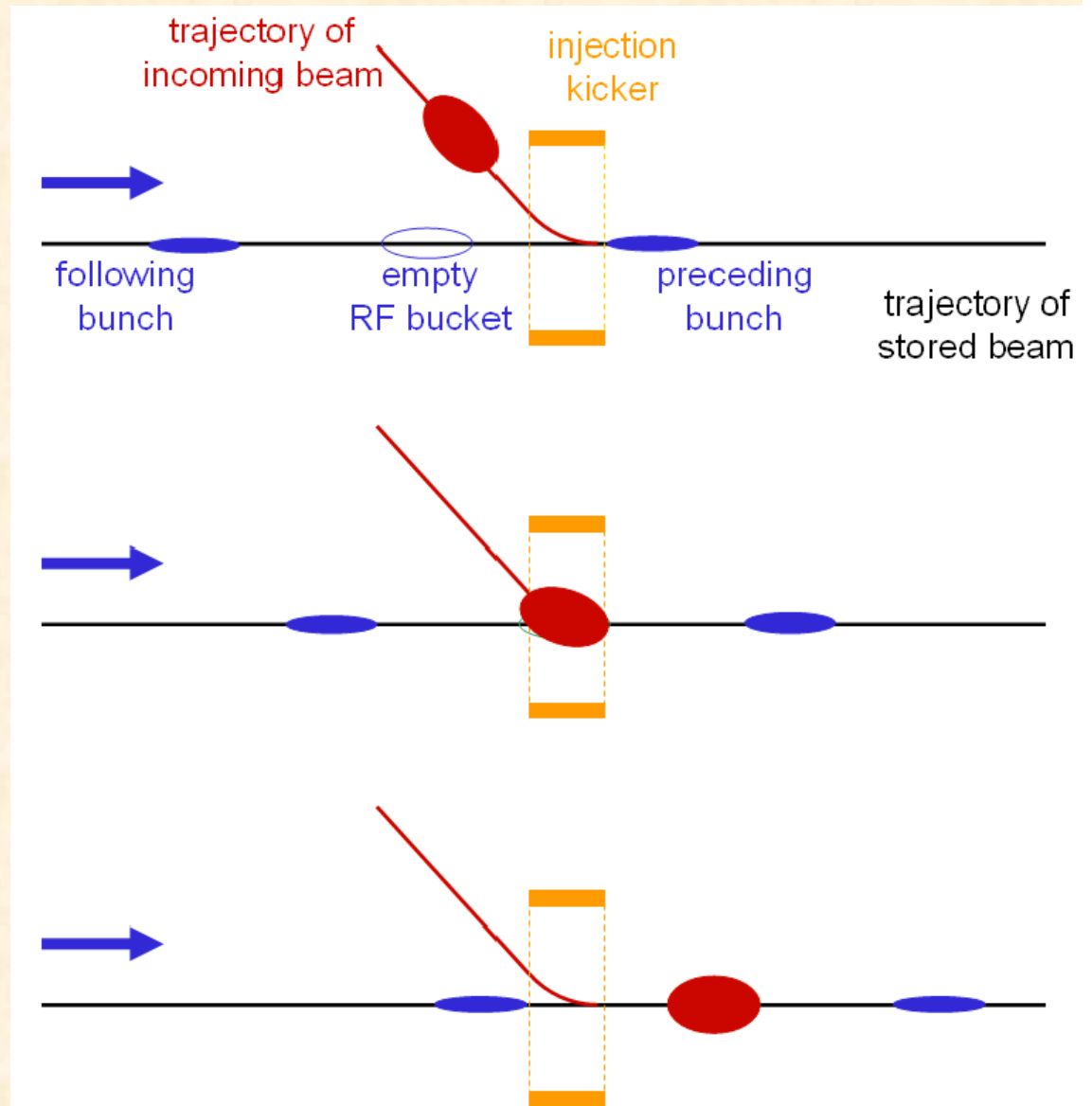


(B tot) (T)

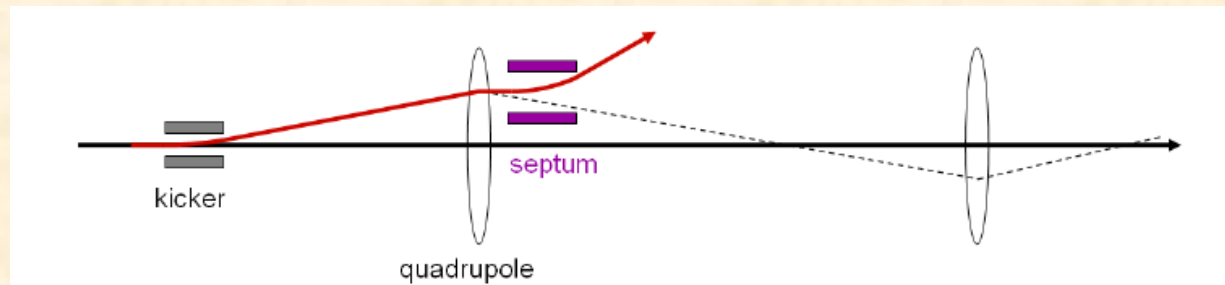


Beam injection/extraction

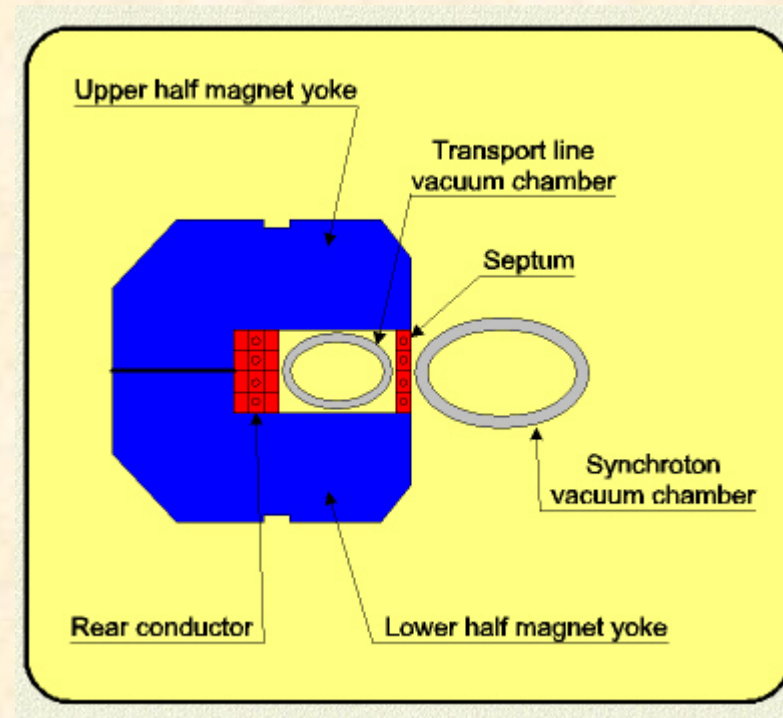
- Injecting (or extracting) particles from a ring is not easy.
- The particles must be inserted on the correct orbit.
- However the deflector must not affect the trajectory of other bunches in the ring (or of the same bunch after one turn).



Septum magnets



- Insertion/extraction are usually done by combining 2 types of deflectors:
 - Ultra fast kickers use an intense electric field to deflect the particles (but beware to ripples)
 - A septum magnet is used to separate neighbouring trajectories (thanks to a magnetic shield).
- This explain why producing H- ions to store protons is interesting.



Magnets summary

- Magnets are used to control particles in an accelerator.
- Dipole magnets are used to change the direction of the beam.
- Quadrupoles are used to control the size of the beam.
- Septum magnets are used to inject extract the beam.

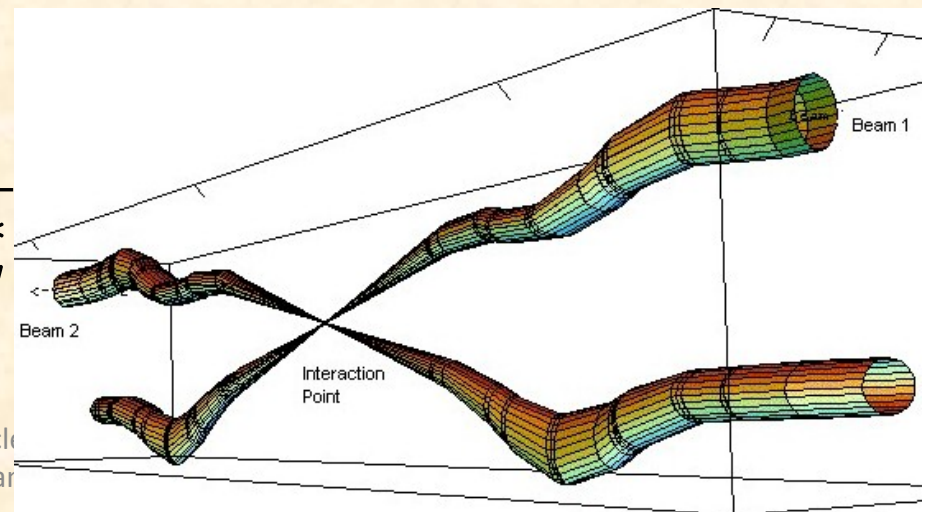


BEAM DYNAMICS

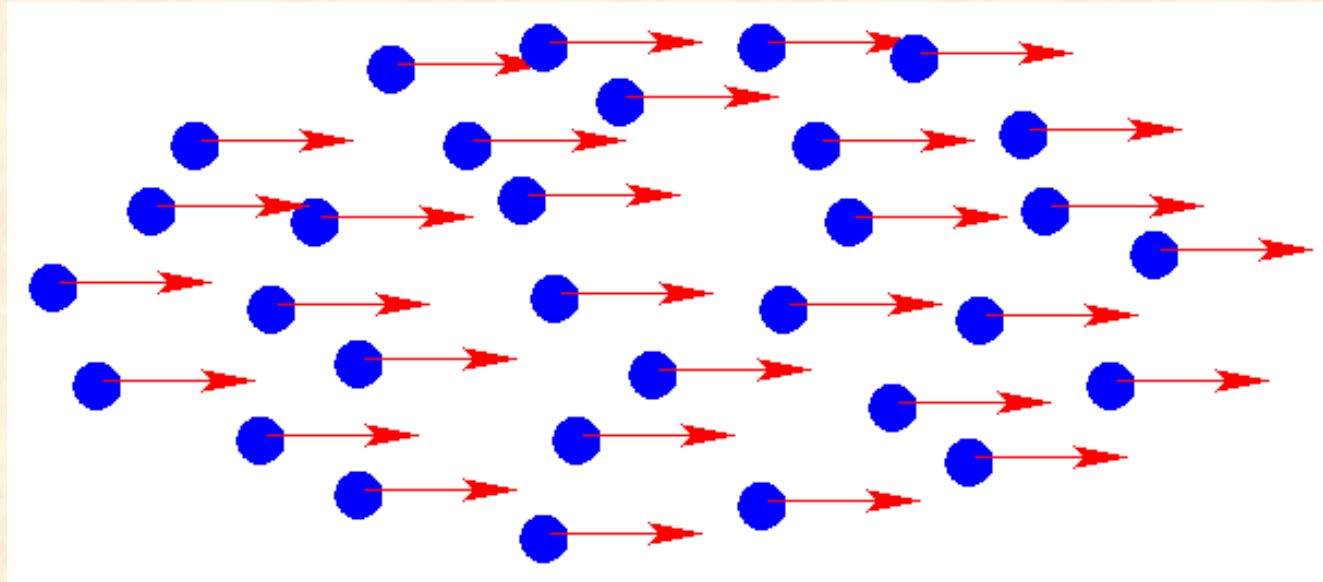
Luminosity

- The luminosity of a collider gives a measure of the number of collisions delivered.
- It is proportional to the collisions frequency and to the bunch charges.
- It is inversely proportional to the beam size.
- However because of the beam size at the interaction point it is often written as a function of two other variables: the emittance and the beta function (magnetic properties of the beam).

$$\mathcal{L} = \frac{f N_1 N_2}{4\sigma_x \sigma_y} = \frac{f N_1 N_2}{4\pi \epsilon_x \beta_x^* \epsilon_y \beta_y^*}$$



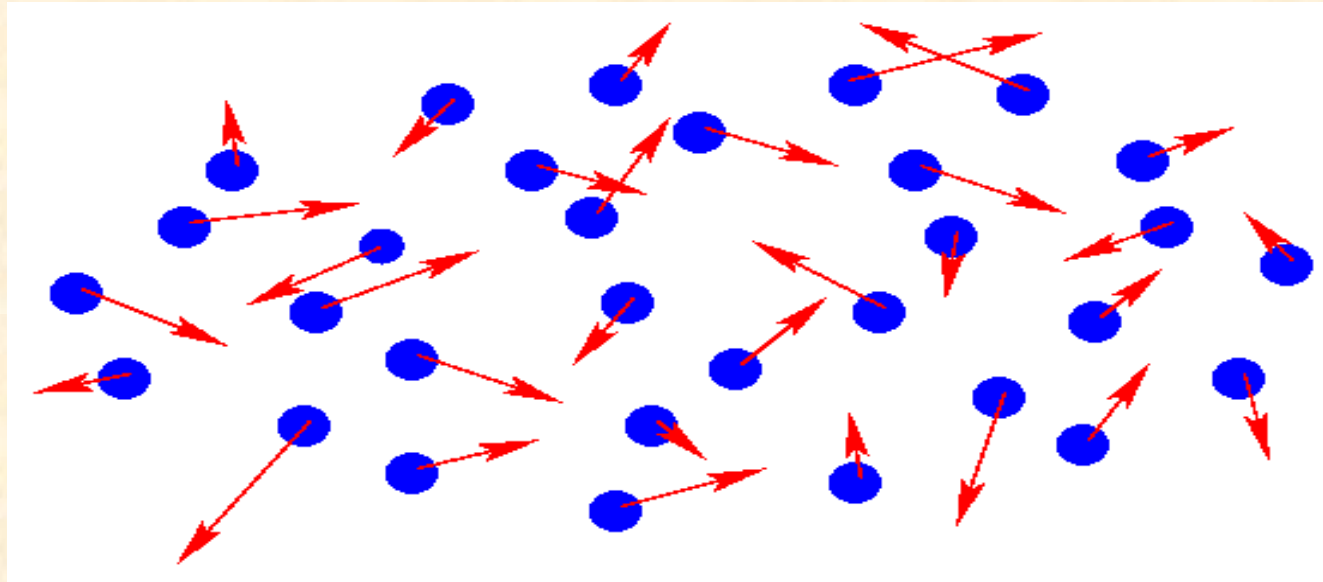
Let's look at a particle bunch



An observer in the laboratory frame looking at a particle bunch will only see particles travelling at the speed of light, apparently all in the same direction.

It is very different if one looks in the bunch's centre of mass frame...

Let's look at a particle bunch



In the centre of mass of the bunch, the particles do not look so well organised...

This should remind you other statistical systems that you have already studied: Gases!

Perfect gas law

- You have studied earlier that a perfect gas obeys the law:
 $PV=nRT$
- V is the volume term ($V=xyz$)
- P is a dynamic term: P , the pressure is proportional to the amount of scattering experienced by atoms as they travel in the volume. It is proportional to the momentum of the gas atoms ($P \sim x'y'z'$).
- Hence it is possible to write that for gas atoms the product of their position by their momentum is expressed by their temperature (*times a constant*).
- We have seen that in the CoM particles look like a perfect gas. The product of their position by their momentum is called the “emittance” of the beam.

6D Trace space

- The position-momentum 6D space is called the trace space.
- To help visualisation the trace space can be decomposed in 3 orthogonal position-momentum planes:

$$xyzx' y' z' = xx'^* yy'^* zz'$$

- It is also often useful to look separately at the transverse and longitudinal planes.

Liouville's theorem

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \sum_{i=1}^d \left(\frac{\partial \rho}{\partial q^i} \dot{q}^i + \frac{\partial \rho}{\partial p_i} \dot{p}_i \right) = 0.$$

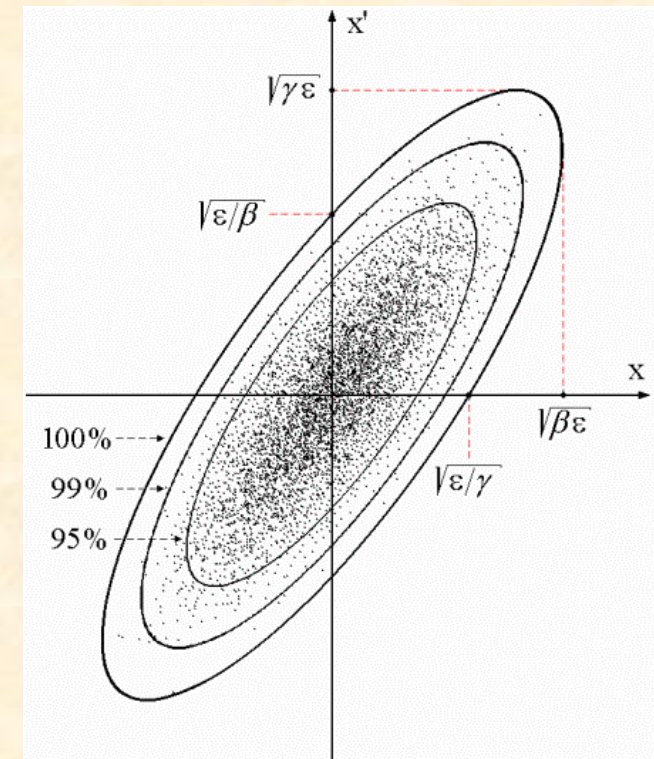
- The volume occupied in the phase space by a system of particles is constant.
- This is a general physics theorem, not limited to accelerators.
- The application of external forces or the emission of radiation needs to be treated carefully.



Joseph Liouville
1809-1882
(source: wikipedia)

Emittance

- We have defined the emittance as the volume occupied by the beam in the trace space.
- Liouville's theorem tells us that such volume must be constant.
- Hence the emittance of a beam is constant (unless external forces are applied).
- The total volume occupied by the bunch in trace-space is usually dominated by a few far-outlying particles.
- Instead of giving the volume occupied by all the particles, it is common to give the volume occupied by 90% or 60% of the particles or to give the RMS emittance.
- The fraction of particles included in the emittance is usually quoted.



$$\epsilon_{90}$$

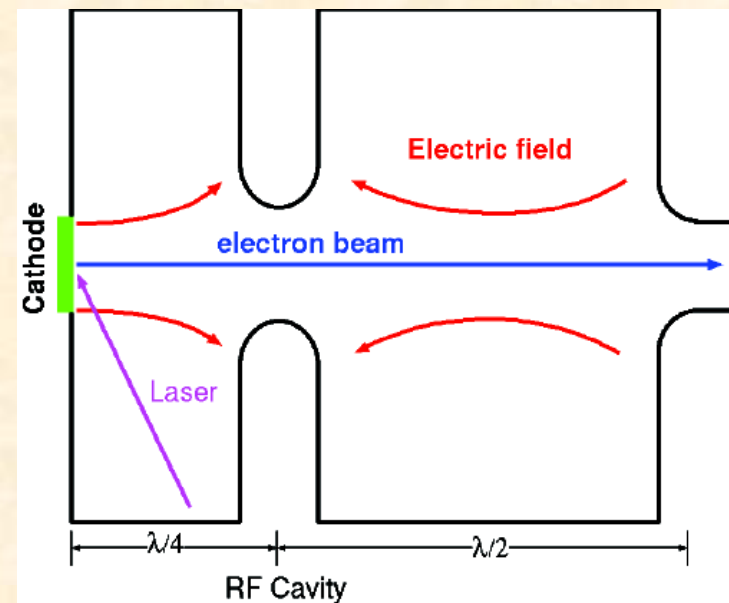
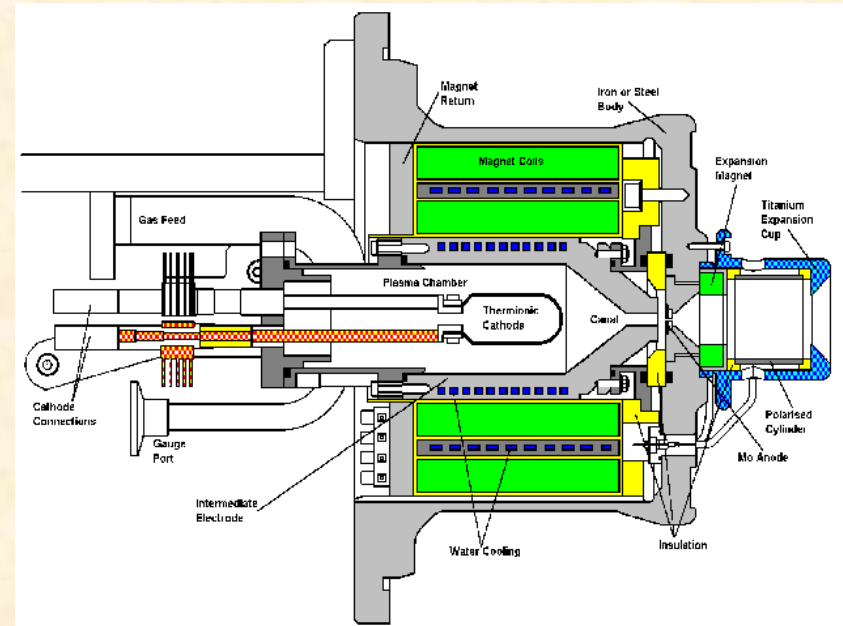
$$\epsilon_{RMS}$$

Quizz:

Gun emittance

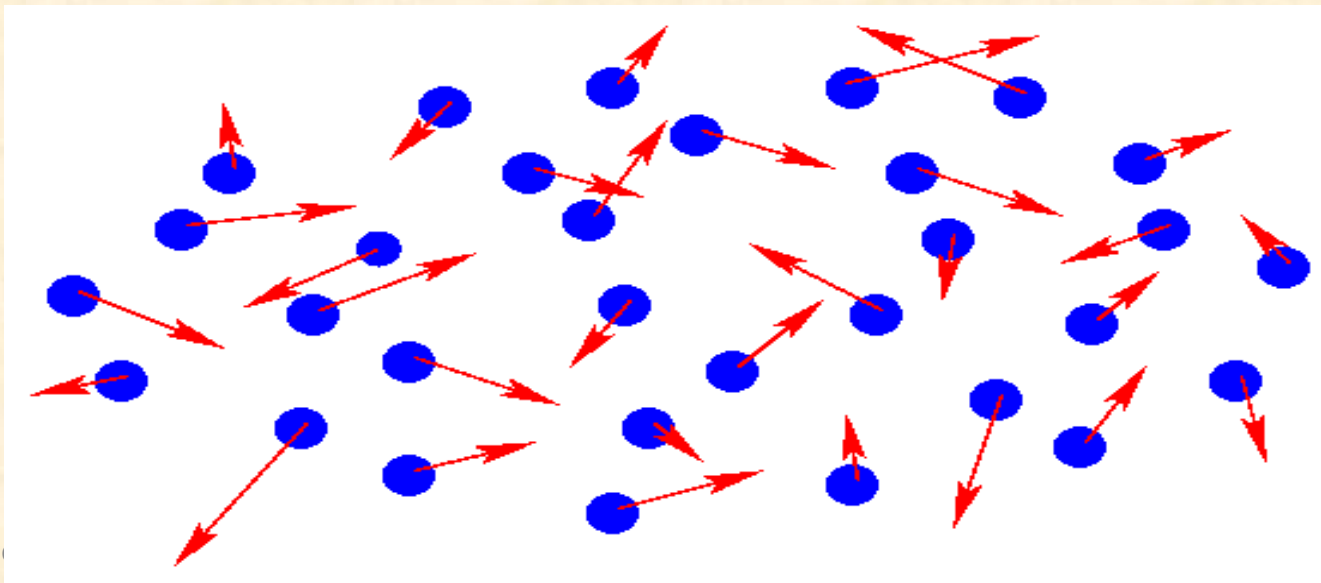
- A particle gun emits particles with a x divergence of 0.5mrad.
- It is fitted with an exit collimator with a hole with a dimension of 1mm in x.
- What is the geometrical x-emittance of the bunch produced by this gun?

- (A) 5mm.mrad
- (B) 1mm. rad
- (C) 500m.nrad
- (D) 50mm.urad



Answer (c)

- The x-emittance at a waist is given by $x \cdot x'$.
- Here we have $x=1\text{mm}$, $x'=0.5\text{mm}$
- That is:
0.5 mm.mrad or 500mm.urad or 500m.nrad.



Quizz

1) Is the emittance of a particle beam an intensive or an extensive physical property?

- a) Intensive
- b) Extensive

2) In which units should the transverse emittance in 1 dimension (x, x') be expressed?

- a) Square metres
- b) Barns
- c) Metre x radians
- d) Kelvin

Answer: (1b)

- Emittance is the product of the beam size by the beam divergence.
- Both are extensive quantities
- The emittance of a fraction of a beam will be smaller than the emittance of the full beam.

$$\epsilon_{90} > \epsilon_{60}$$

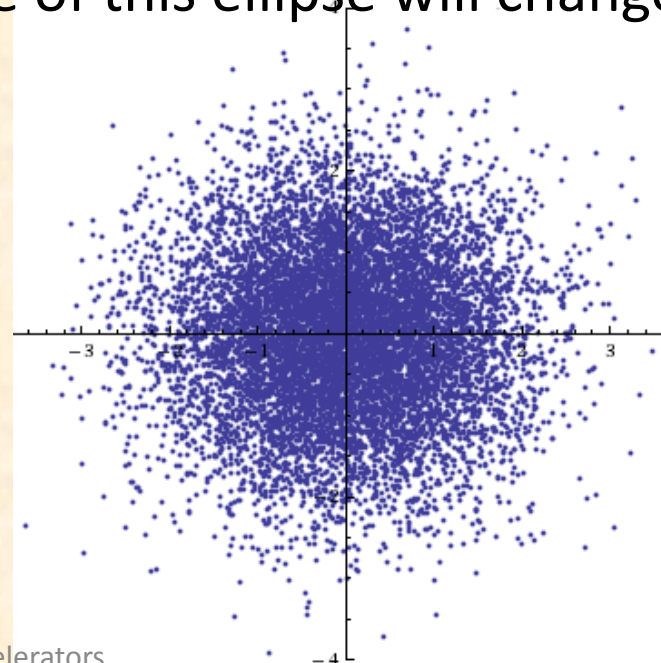
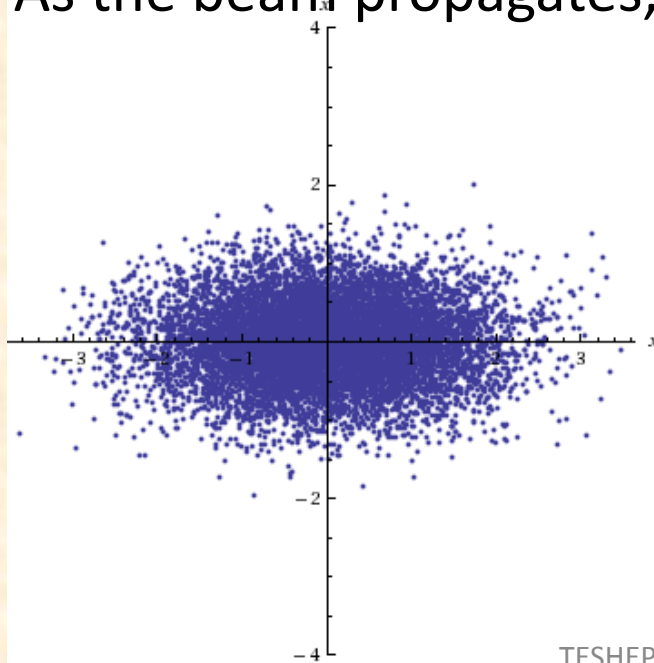
- This property is used in particle accelerators: the emittance of a beam can be improved by trimming it.

Answer: (2c)

- The size of the beam is expressed in (milli)metres.
- The divergence of the beam can be expressed either in (milli) radians or in eV.
- Depending on conventions a factor π may be added.
- A typical emittance for an electron linac with a thermionic gun is mm.mrad.
- A synchrotron can go below nm.mrad.
- Do not forget that $1 \text{ mm.mrad} = 1 \mu\text{m.rad}$!

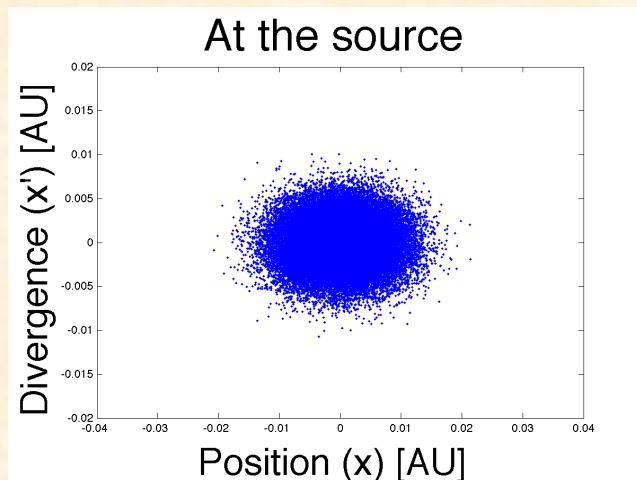
Emittance ellipse

- A random Gaussian distribution of particles forms a straight ellipse.
- By choosing the right set of coordinates this ellipse can be transformed in a circle (do not forget that the two axis are orthogonal!).
- As the beam propagates, the shape of this ellipse will change.

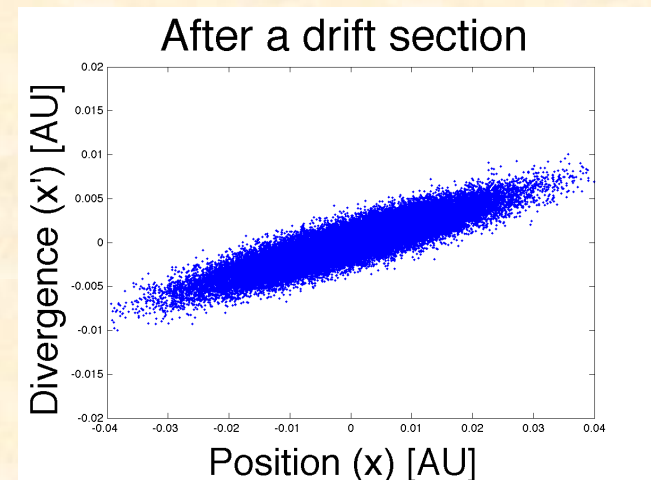


Beam drift

- When the beam “drifts” that is, propagates in space, over a length L with no external forces applied:
 - The momentum of the particles is constant
 - The position changes by the momentum times L .
- Hence, the emittance ellipse is sheared.

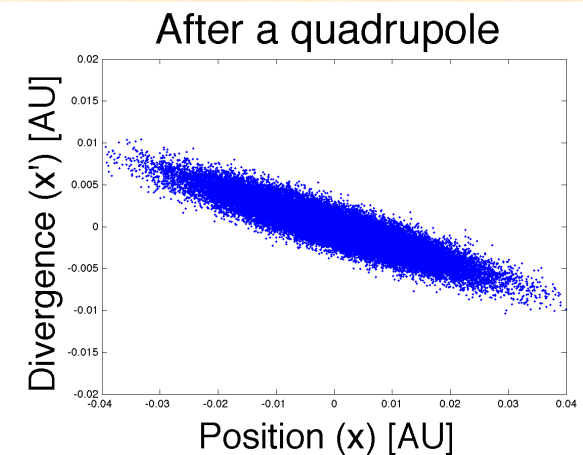
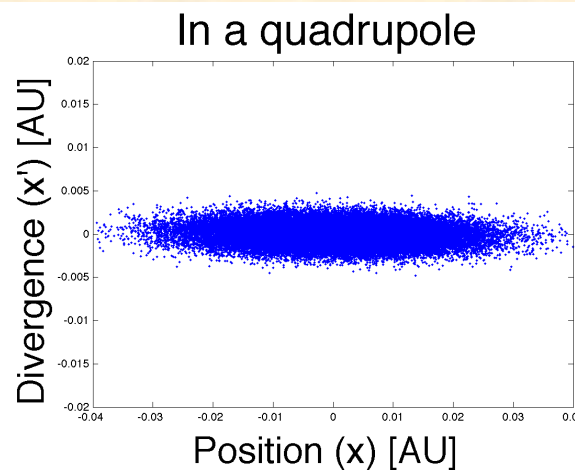
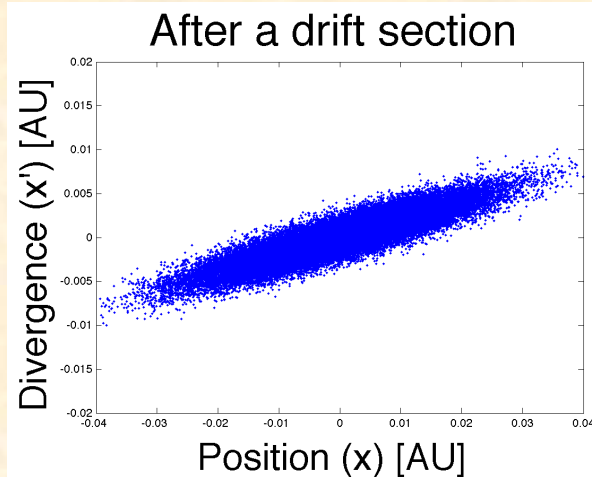


$$x'_2 = x'_1$$
$$x_2 = x_1 + Lx'_1$$



Focussing

- In a focussing section (typically a quadrupole), in the thin lens approximation:
 - The position of the particles is not affected
 - The momentums are reversed, hence a waist (at which all $x'=0$) is formed.
- The ellipse is unsheared and then sheared in the other direction



$$x_2 = x_1$$

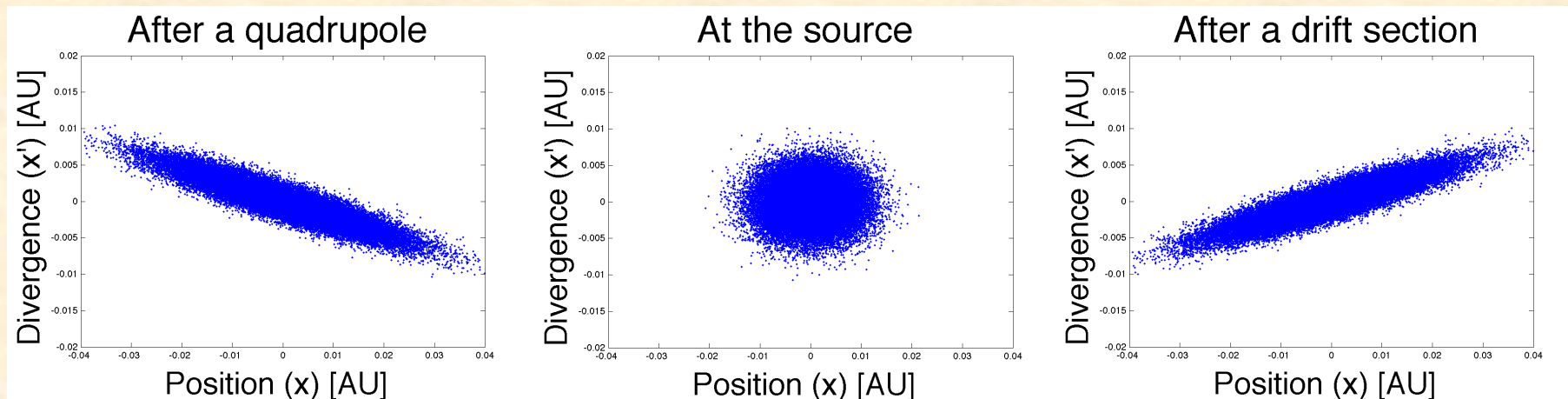
$$x'_2 = 0$$

$$x'_3 = -x'_1$$

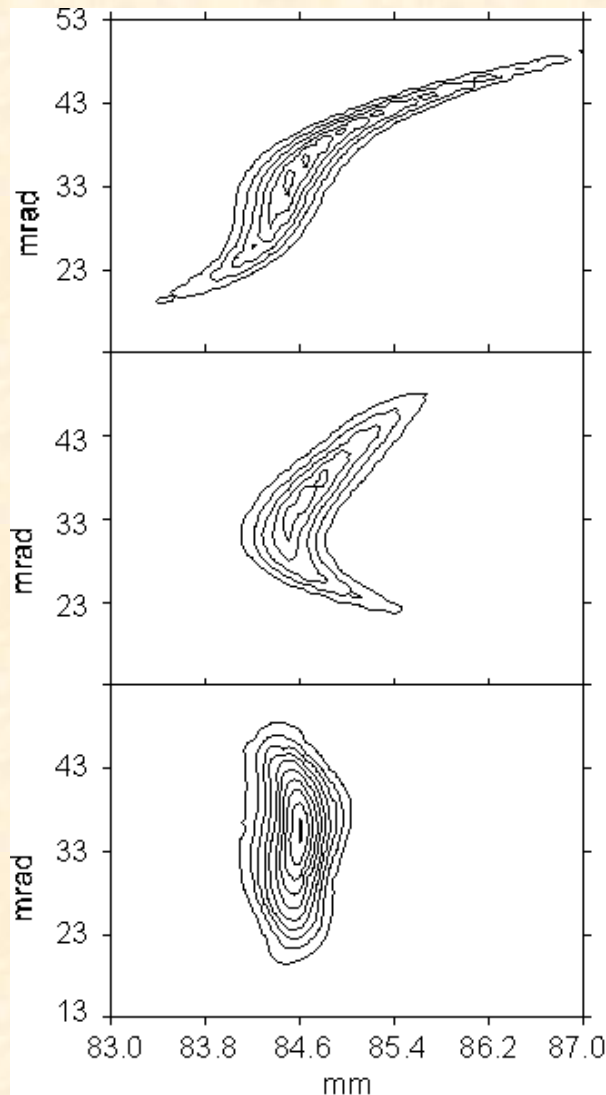
$$x_3 = x_1$$

Beam waist

- After the focussing section the beam will drift again, decreasing the shearing of the emittance ellipse.
- At some point the momentums will again average to 0, the beam will be forming a waist.
- At the waist the shearing of the emittance ellipse flips and starts increasing again.
- The beam size is the smallest at the waist.



Non linear effects



Source:TRIUMF

Nicolas Delerue, LAL Orsay

- Magnet non linearities will increase the deformation of the emittance ellipse.
- Higher order magnets are required to correct such deformations (octupole and sextupole).
- Example of emittance measured at an accelerator in Canada.

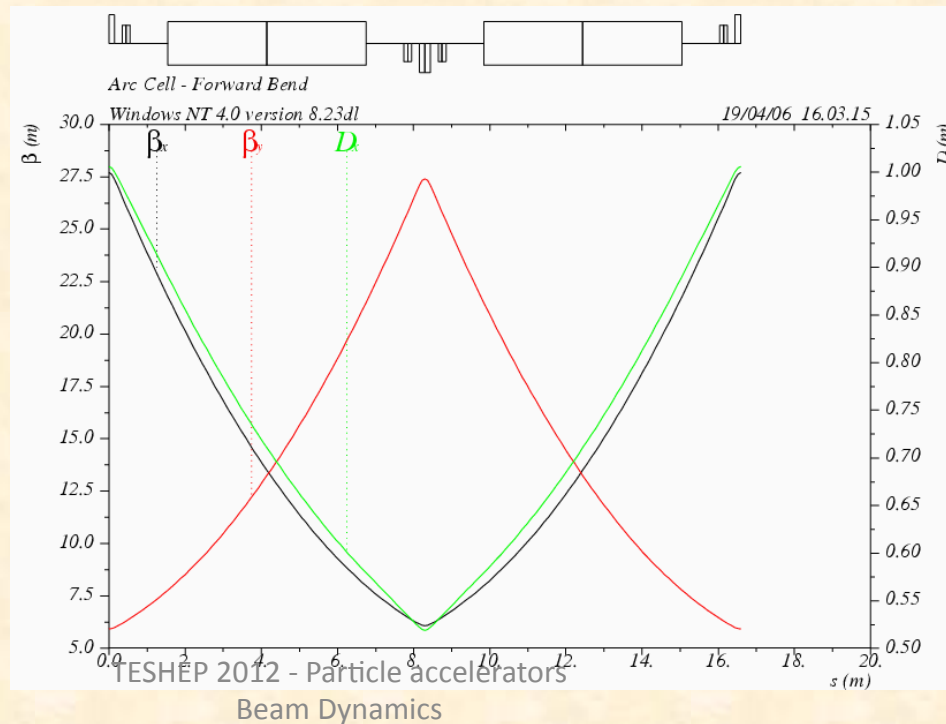
Beta function

It is convenient to define the “beta function” to relate the beam size to the emittance.

$$\beta = \frac{\sigma^2}{\epsilon}$$

$$\sigma = \sqrt{\beta \epsilon}$$

$$\sigma_{RMS} = \sqrt{\beta \epsilon_{RMS}}$$

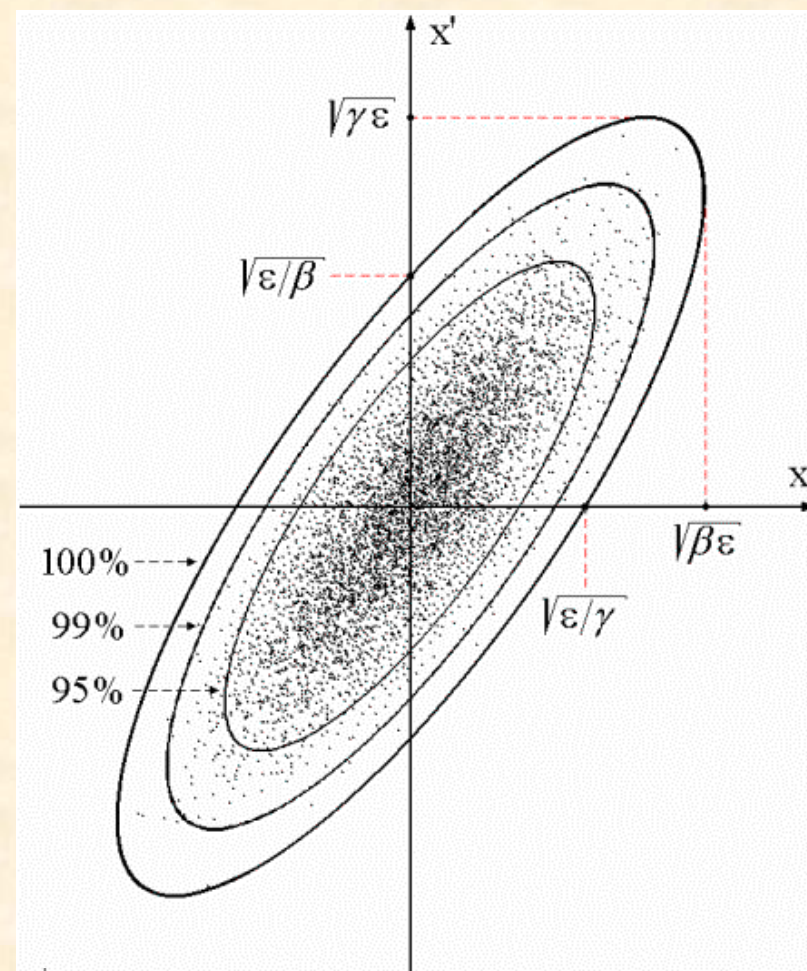


Beam parametrisation: Twiss parameters

$$\sigma = \sqrt{\epsilon(\beta_0 - 2\alpha_0 s + \gamma_0 s^2)}$$

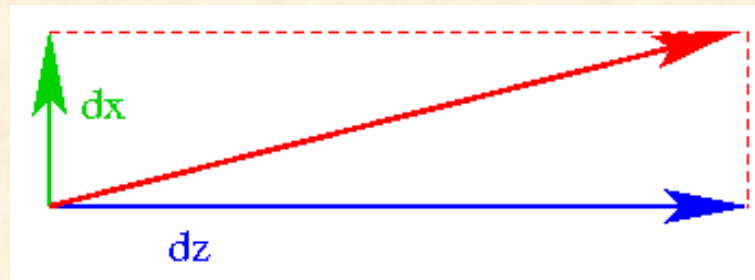
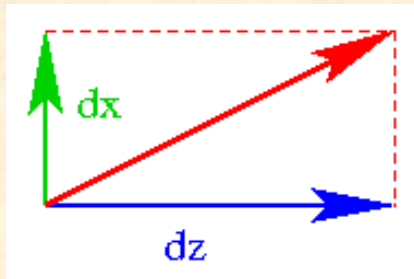
- The emittance ellipse can be described using the Courant-Snyder representation (Twiss parameters).
- Like any other physical system in which elements travels in straight line, the beam envelope forms an hyperbola.

$$\sigma = \sqrt{\epsilon(\beta_0 + \frac{s_w^2}{\beta_0})}$$



Acceleration

- When the beam is accelerated, its longitudinal momentum is increased,
- But the transverse momentum remains the same.
- Hence the beam divergence decreases.



- Accelerating the beam leads to a reduction the volume occupied in phase space.
- This reduction is proportional to the increase of the relativistic gamma.
=> The beam size is reduced when the beam is accelerated!

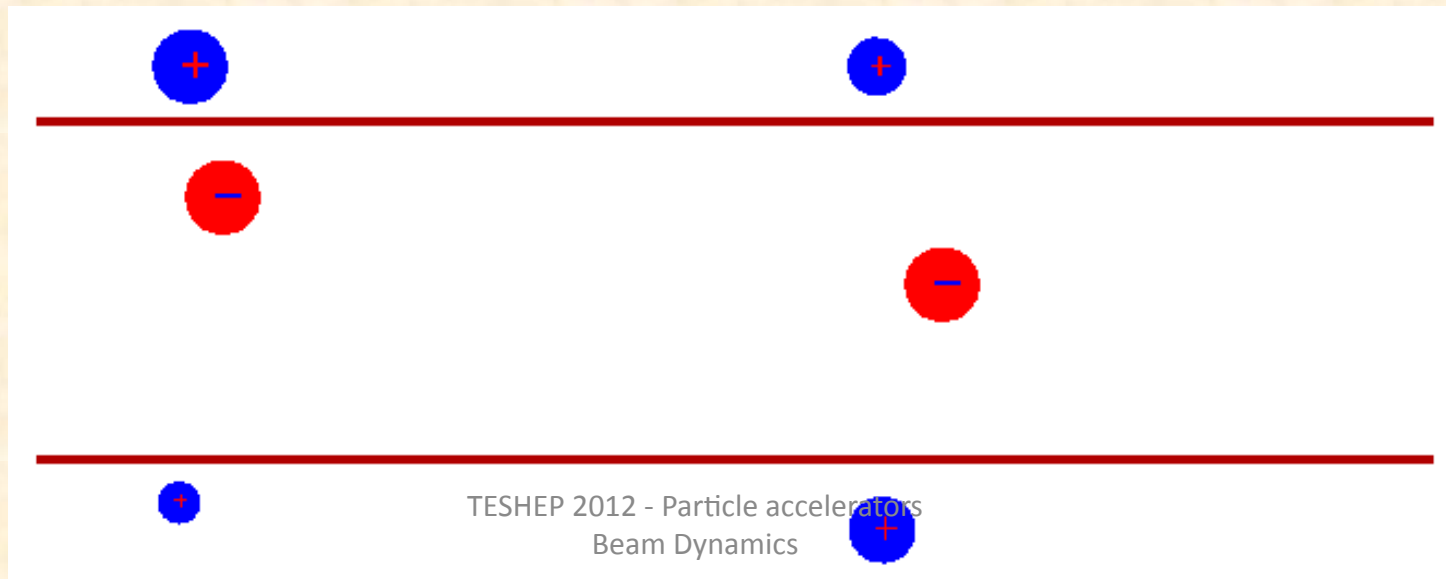
Normalised emittance

- It is convenient to define the normalised emittance of a beam: it is the volume of phase space occupied by the beam multiplied by gamma.
- The actual volume of phase space occupied by the beam is called the geometric emittance.
- The normalised emittance of a beam is constant under acceleration.

$$\epsilon_N = \gamma \epsilon_{Geometric}$$

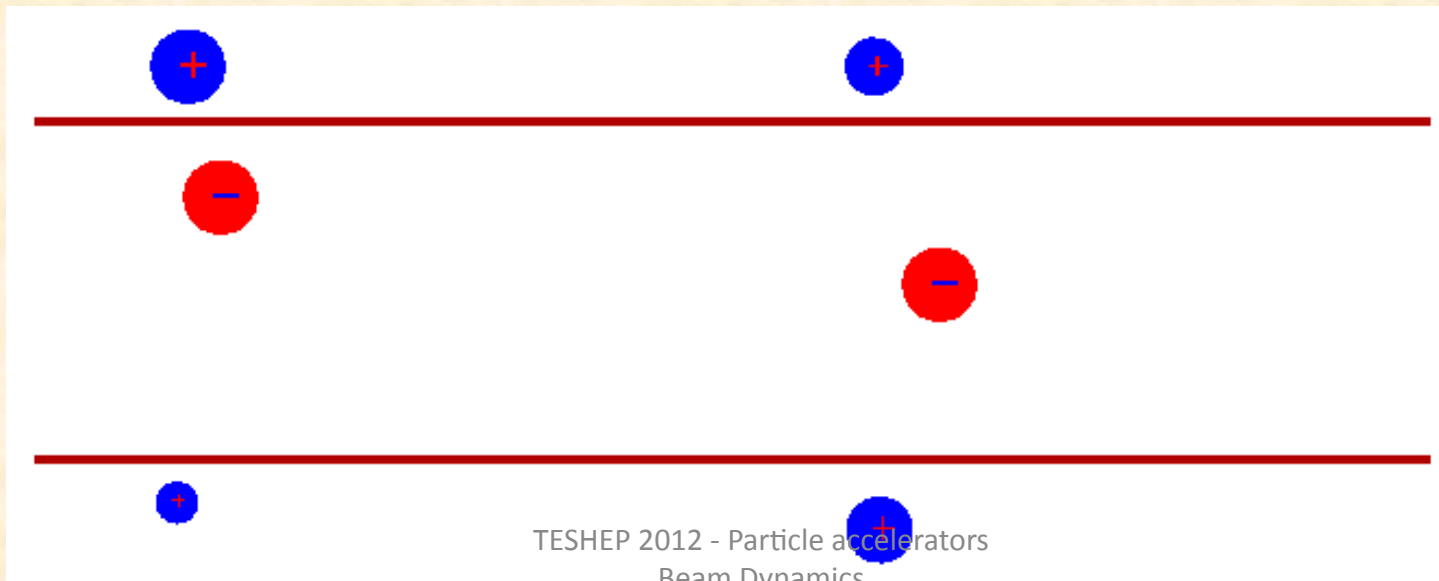
Impedance issues

- Charged particles travelling near a conductor induce image charges (induced current).
- This image current dissipate power in the beam pipe (Joule effect).
- The smaller the beam pipe, the higher the induced charge and thus the highest the losses.
- The impedance of a beam pipe must be carefully controlled!



Quiz

- In the example below, which particle will create the more induced current on the beam pipe?
 - a) The particle on the left hand-side
 - b) The particle on the right hand-side
 - c) Both will induce the same current



Answer (c)

- Apply Gauss's law:
$$\oint_{\partial\Omega} E \cdot dS = \frac{Q(V)}{\epsilon_0}$$
- The total charge going through the beam pipe is the same for both particles.
- Hence the total current induced on the beam pipe by both particles will be the same.

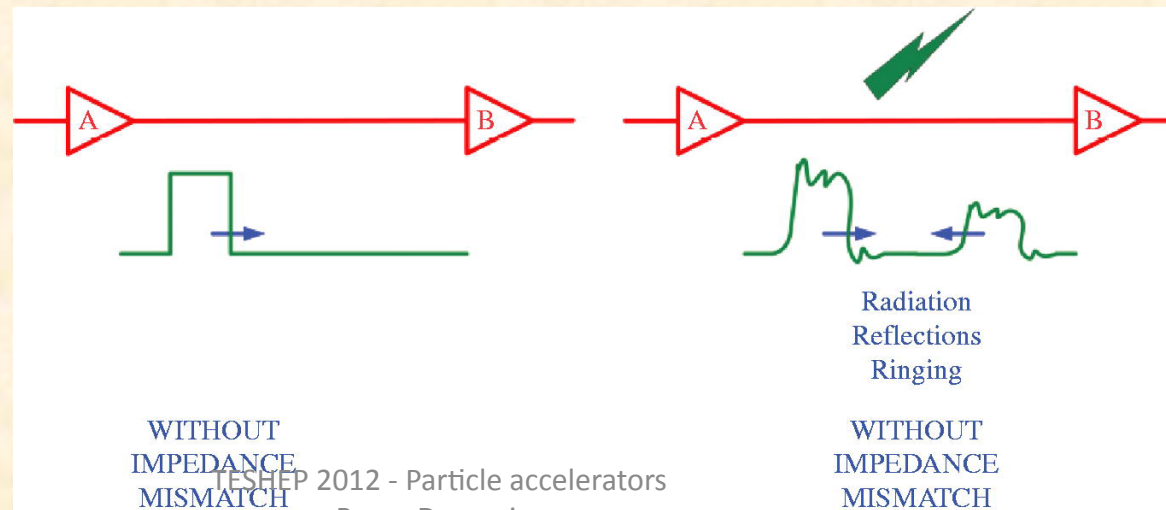
Wake field issues

- Electrons produce an electromagnetic wave behind them.
- This can be compared to the wake of a boat and is called wake field.
- Imagine what would happen if there was a second surfer on the picture below...
- How good is the wake for the walls of the canal?
It is not good for the beam pipe either!



Impedance matching

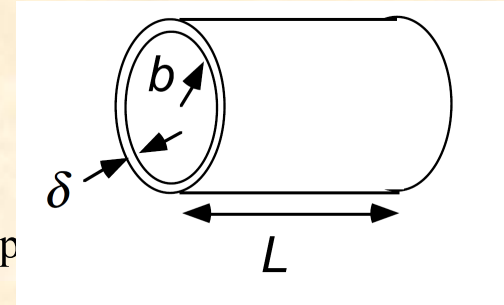
- In a electronic circuit an impedance mismatch will result in a reduced transmission at the interface.
- The same is true in an accelerator: an impedance mismatch is likely to induce a reflective wave at the interface.
- This will induce a loss of power and an emittance increase.
- In a synchrotron the impedance of all beam pipe elements is carefully controlled.
- This is less important in a transfer line where the beam passes only once.



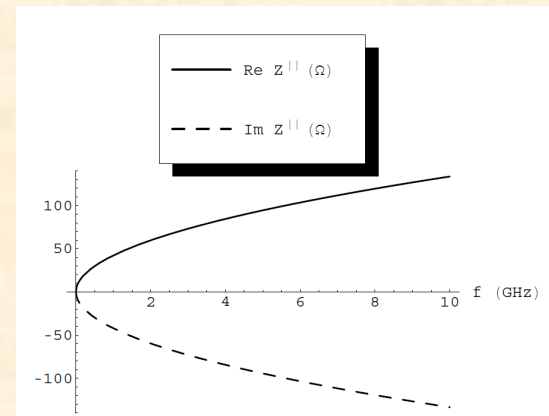
Impedance

- Let's consider a section of beam pipe.
- The impedance depends on the skin depth and the area of the pipe.
- C = ring circumference
- This impedance will induce a wake field, intense behind the beam but with a long tail
=> long distance effect for the following bunch.

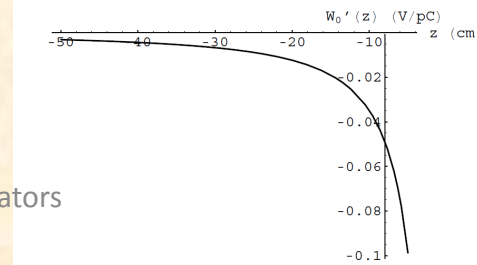
$$\delta = \sqrt{\frac{2}{\sigma\mu\omega}} \quad (\text{skin dep})$$



$$\frac{R_{wall}(\omega)}{L} = \frac{1}{\sigma A} = \frac{1}{\sigma 2\pi b \delta} = \frac{1}{\sigma 2\pi b \delta} = \frac{1}{2\pi b} \sqrt{\frac{\mu\omega}{2\sigma}}$$



$$W'_0(z) = -C \frac{c}{4\pi b} \sqrt{\frac{c\mu}{\pi\sigma}} \frac{1}{\sqrt{|z|}^3}$$



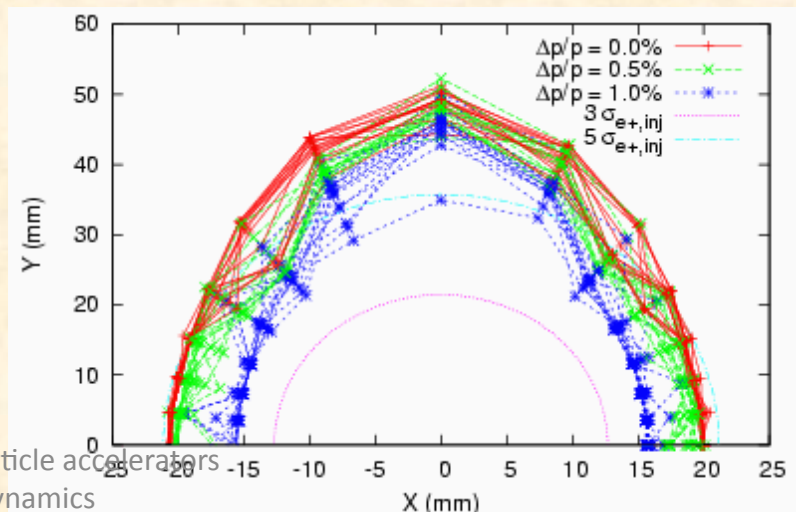
Beam size and magnet aperture

- One of the motivations for keeping the beam small is that magnets have a limited aperture.
- The larger the aperture the more difficult it is to keep a uniform field (and the bigger/more expensive the magnet is).
- It is recommended to have a beam pipe 5 times larger than the RMS size of the beam.



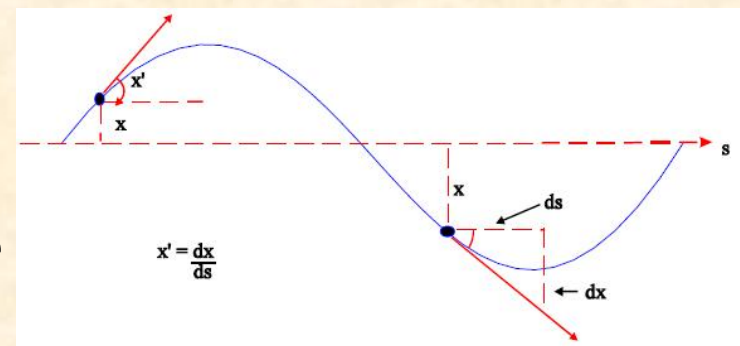
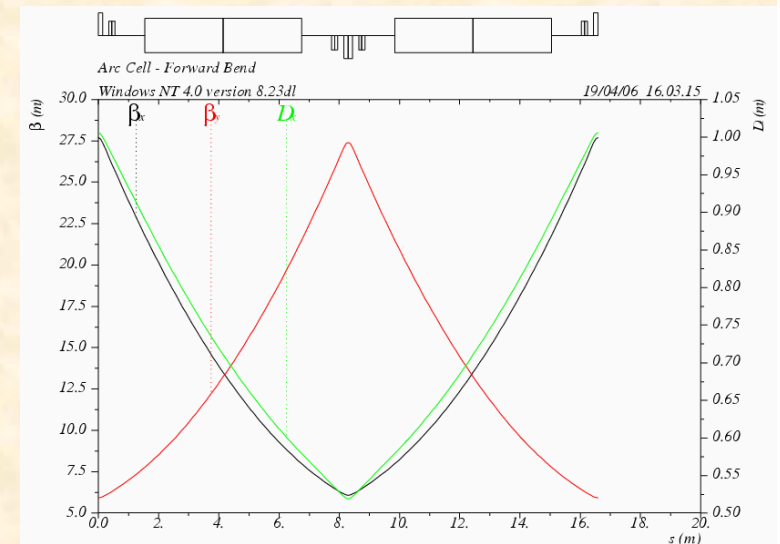
Magnet aperture

- Clipping, wake field and impedances issues lead to poor accelerator performances.
- To avoid these effects the trajectory of the beam in the accelerator must be simulated.
- Tracking software are used to do this: they study how particles move from one location to the next.
- Instead of tracking each particle individually it is enough to track the envelope of the beam.



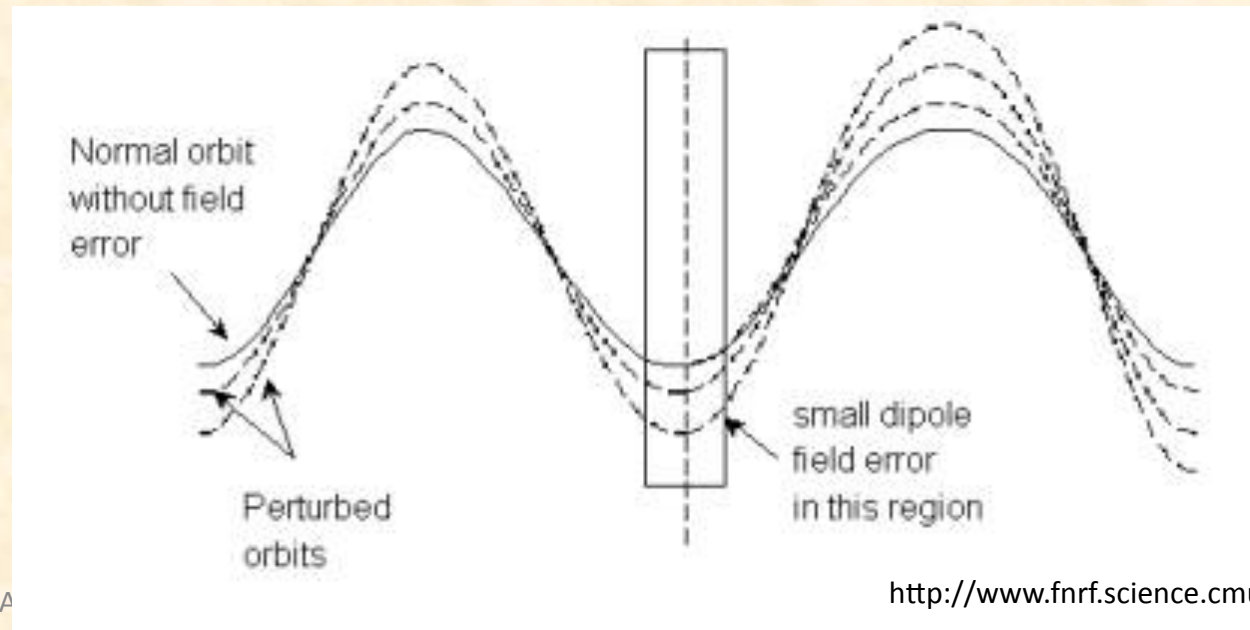
Betatron oscillations

- We have seen that as it travels along the lattice the beam is focussed alternatively in both planes.
- For individual particles this leads to oscillations called “betatron oscillation”.
- This occurs in both planes at the same time.
- If the particles perform an integer number of betatron oscillations in one turn, they come back at the same position turn after turn.
- Be careful, you should not confuse the number of lattice periods with the number of betatron oscillations.



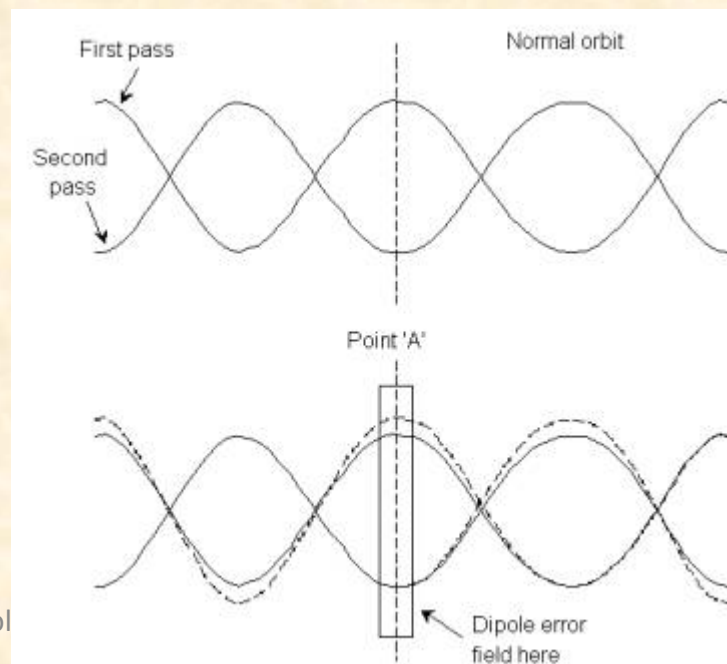
Integer betatron oscillations

- If one of the magnets in the accelerator has a field error a particle coming turn after turn at the same position will accumulate the effect of this field error.
- After a large number of turns (millions...) what was initially a very small error can lead to large orbit changes and eventually to the loss of the beam.
=> avoid integer betatron oscillations...

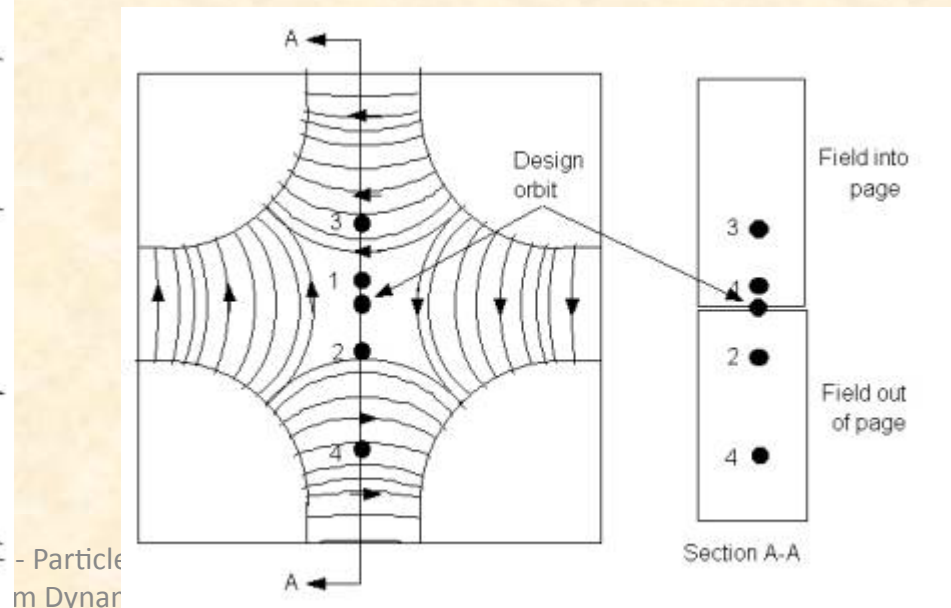


Half-integer betatron oscillations

- Using an optic with an half-integer number of oscillations avoids first order problems.
- However a similar effect may accumulate every other turn...

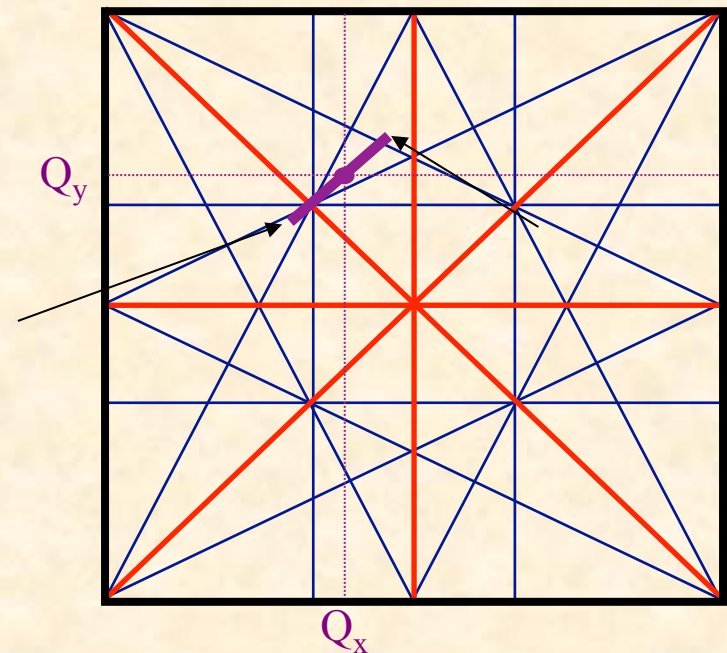


Nicol



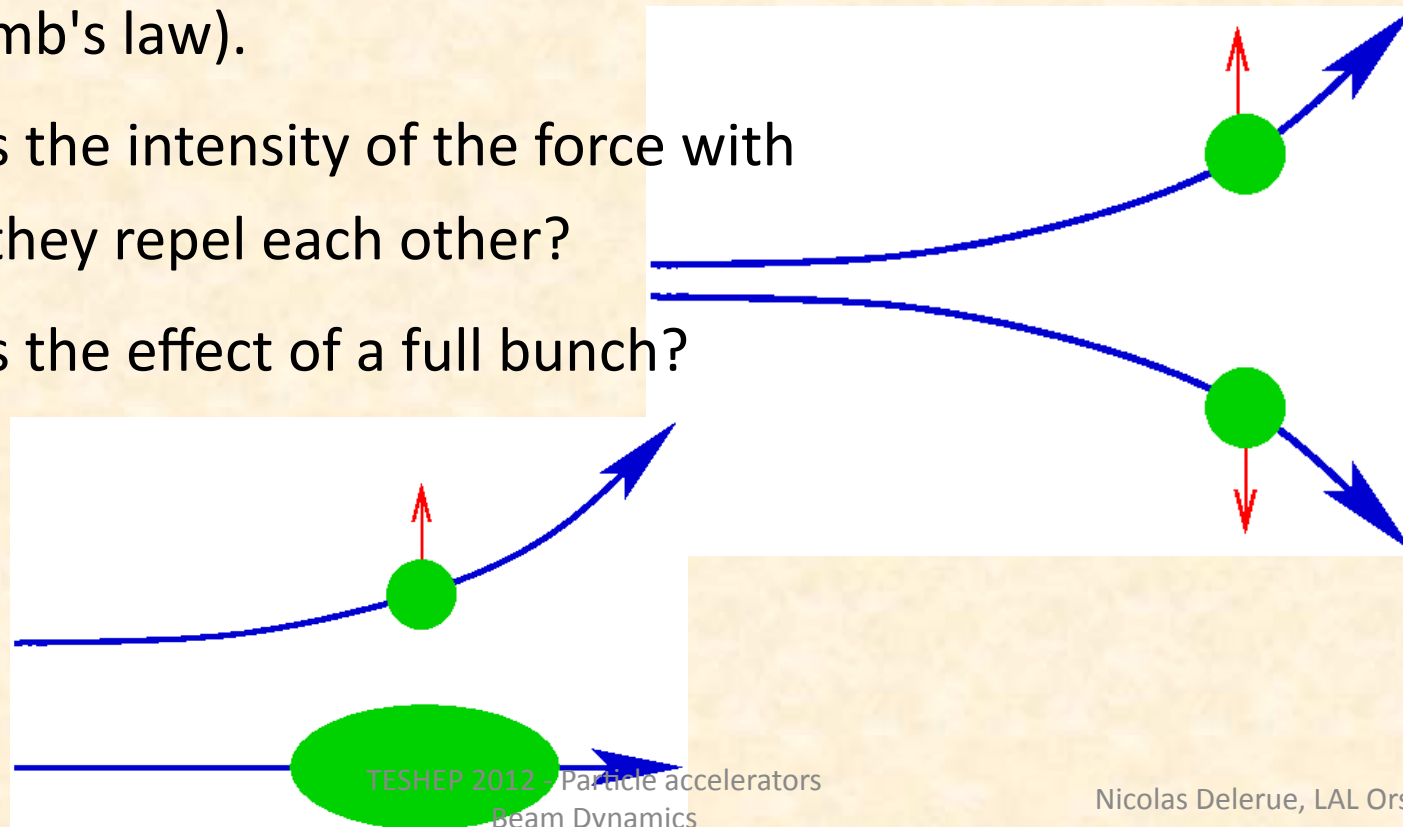
Tune

- To avoid cumulative effects in the accelerator over a large number of turns all rational numbers should be avoided for betatron oscillations in both planes
=> choose an irrational number!
- Choosing an incorrect tune can significantly increase the particle loss rate.



Space-charge effect

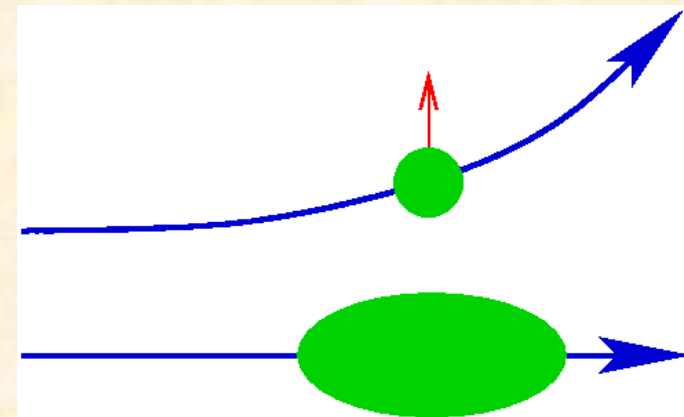
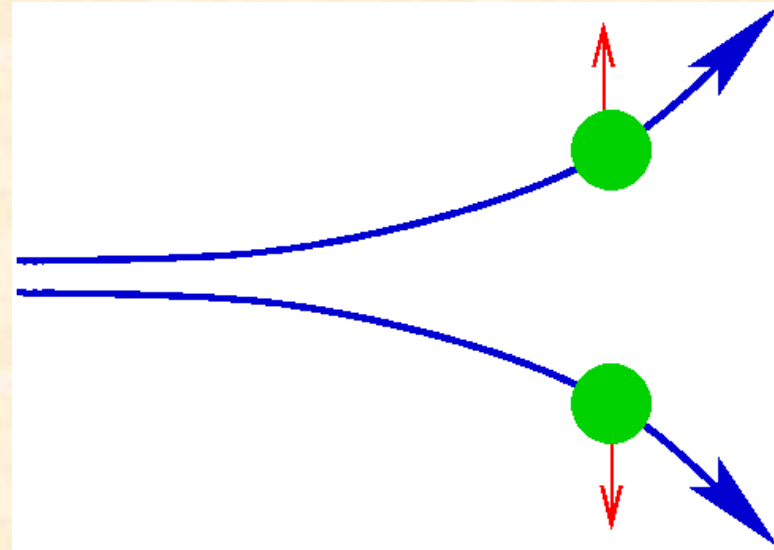
- Now let's consider two particles with similar charges travelling in the same direction.
- Due to their charge these particles will push each other away (Coulomb's law).
- What is the intensity of the force with which they repel each other?
- What is the effect of a full bunch?



Coulomb force between two electrons

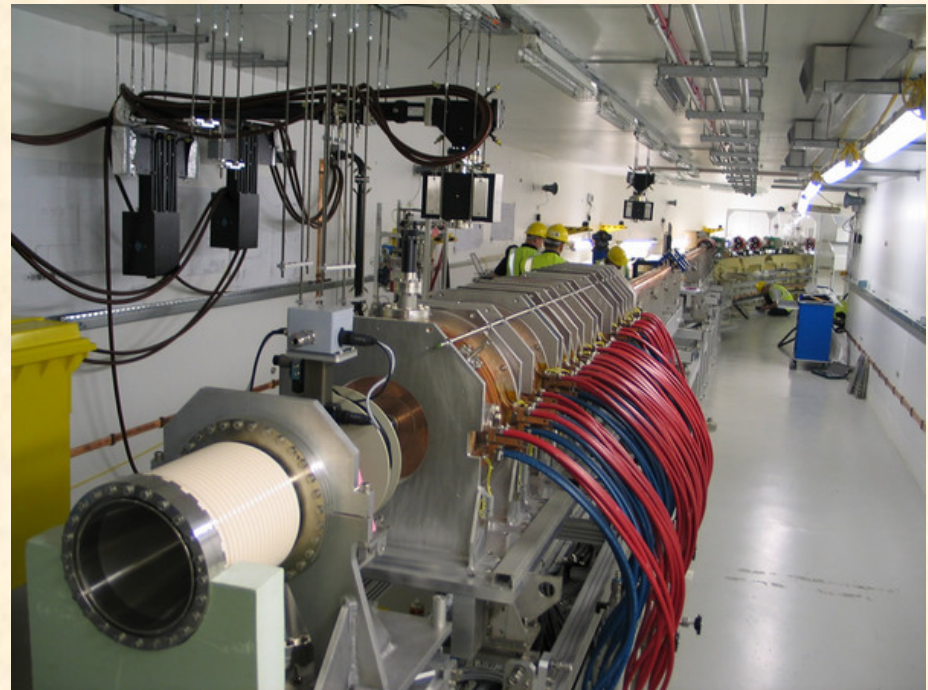
$$f = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{d^2}$$

- Assume $d=1\text{ micrometre}$.
- $f=2 \cdot 10^{-16}\text{N}$
- This may look small but an electron is not very heavy
- $f/m_e=2.5 \cdot 10^{14}\text{N/kg}$
- This force is very intense on the scale of the electrons.
- Typical charge in a bunch:
 $\sim 1\text{nC} = 1.6 \cdot 10^{10}$ electrons

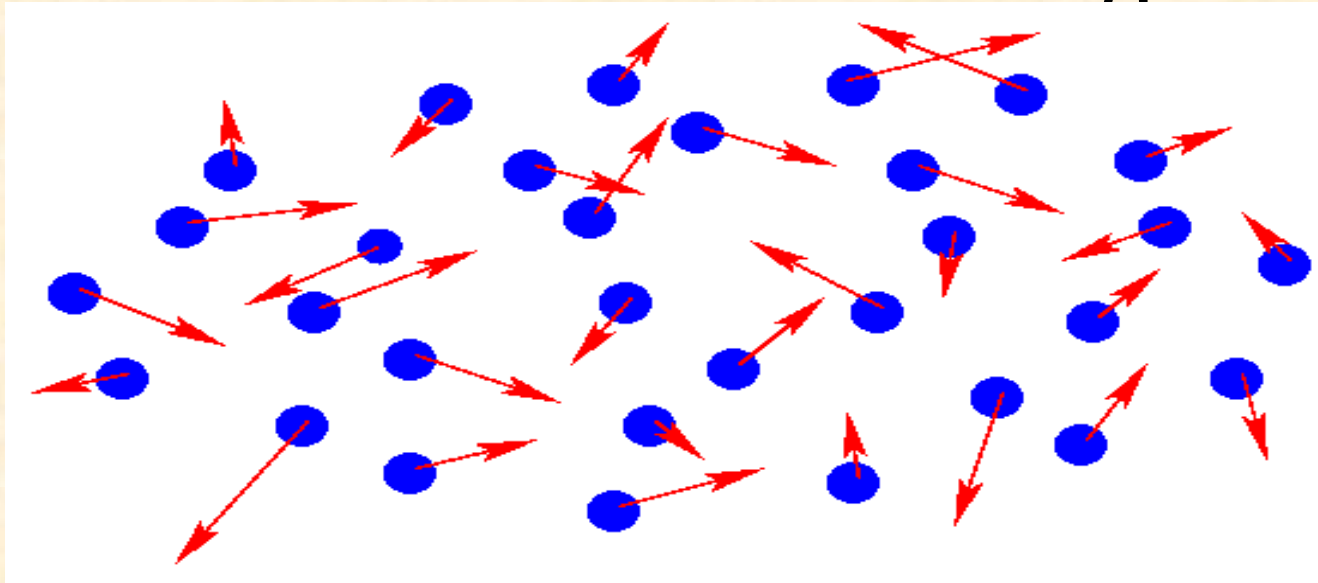


Compensating space-charge

- At low energy, space-charge is a limitation of accelerators. Compensating devices have to be used to mitigate its effects.
- At high energy the “rigidity” of the beam makes space-charge less of an issue but it can not always be ignored.



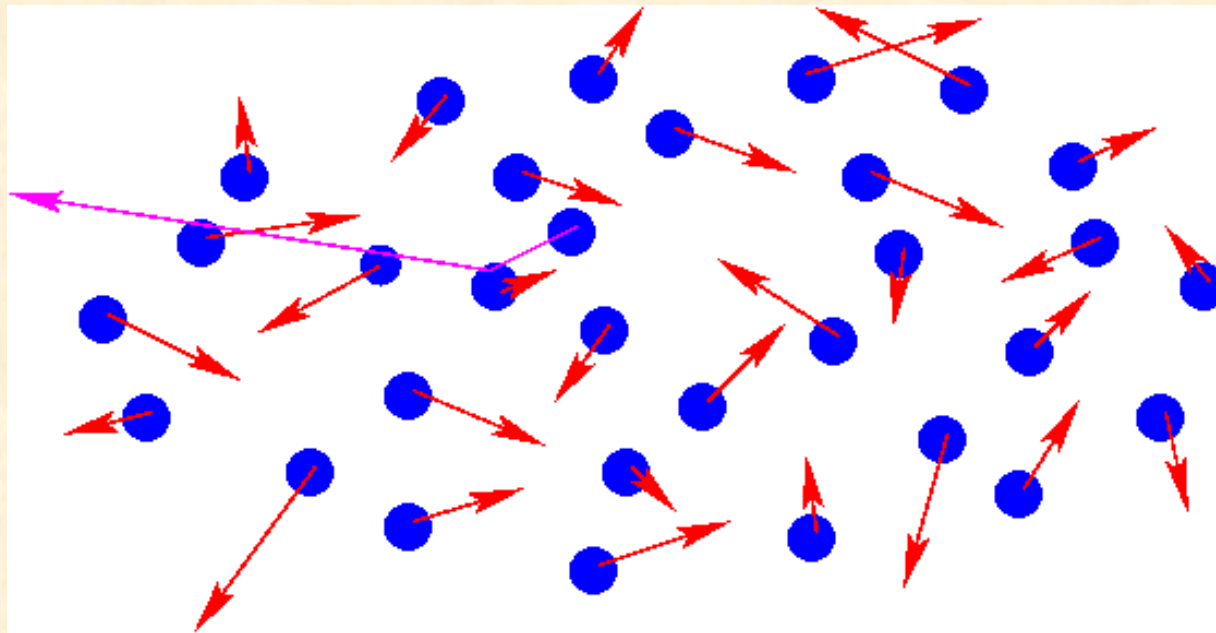
Intra-beam scattering



- We have seen that inside the beam the particles behave somewhat like a gas.
- Coulomb collisions do occur between the particles.
- These collisions lead to a momentum transfer between the particles and thus an emittance coupling and emittance growth.
- Beams with a larger emittance will experience more IBS.

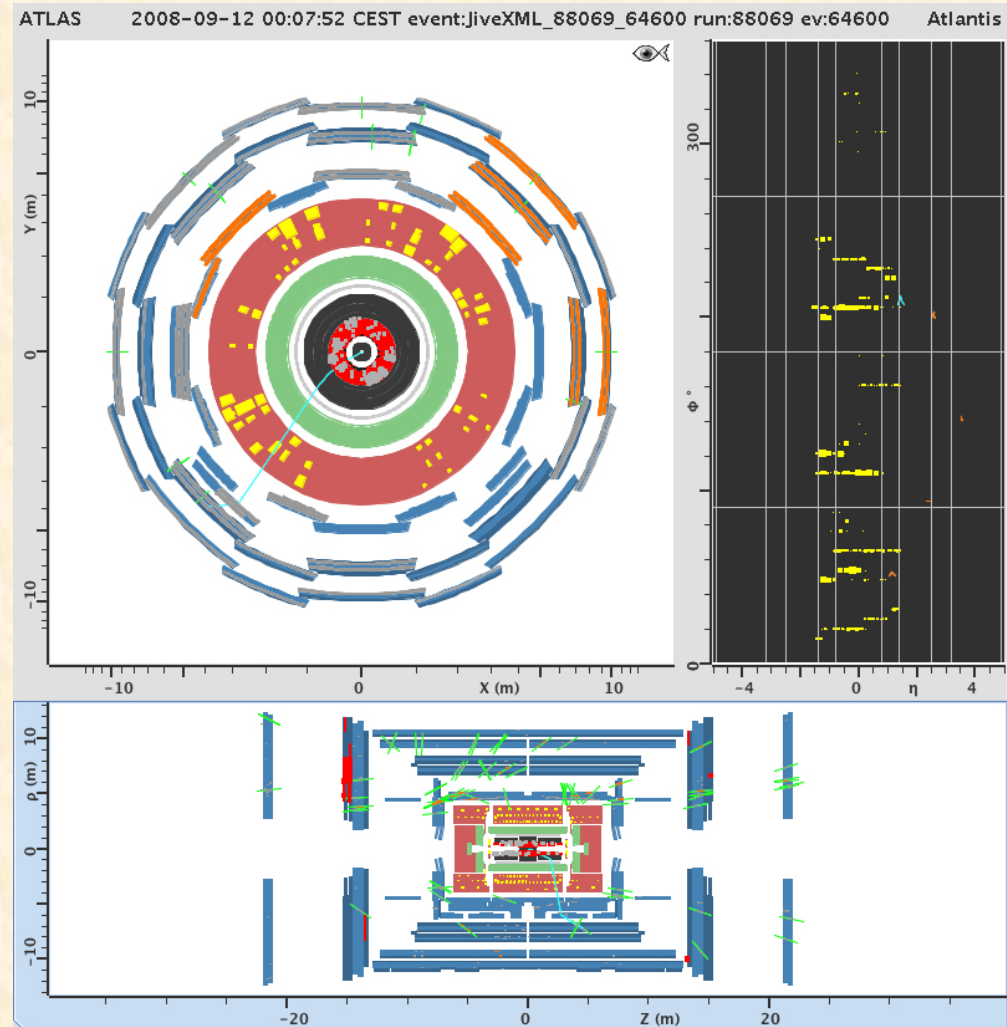
Touschek effect

- In addition to Coulomb scattering, hard scattering can also occur.
- In most cases this will lead to one particle being pushed out of the acceptable beam orbit and thus being lost soon after.
- Touschek scattering occurs in high current beams.



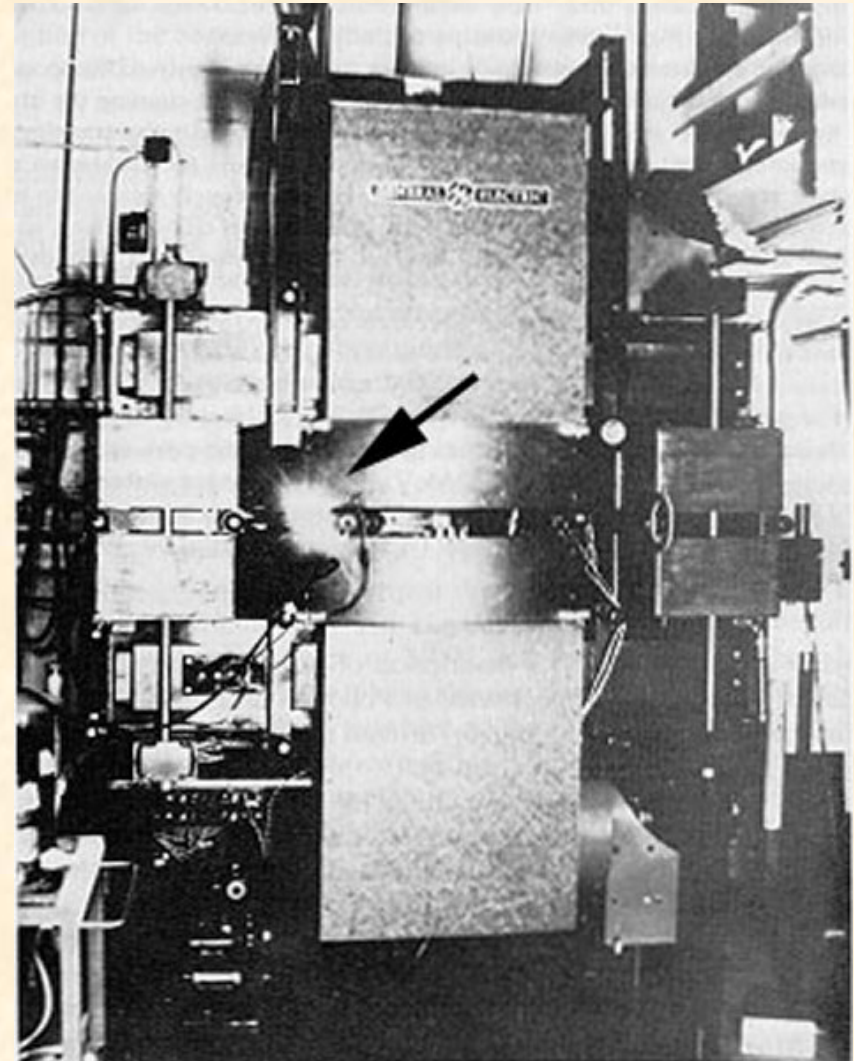
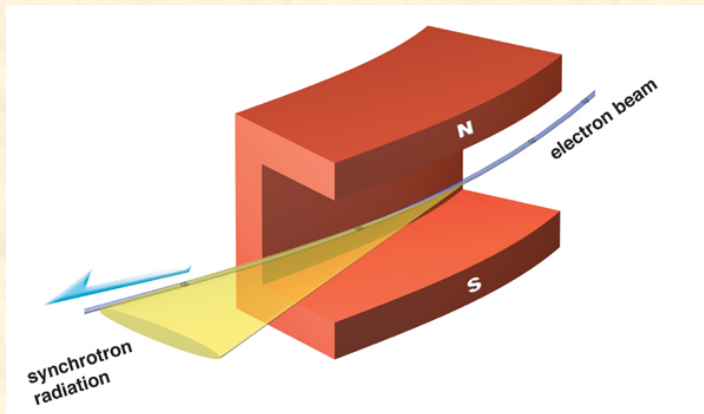
Halo

- Charged particles attract particles with the opposite charge.
- This creates a halo of particles with opposite charge around the beam.
- This halo of particles is not confined to the beam and thus it can damage equipment outside the beam pipe also.
- It is a source of beam instabilities and can lead to significant emittance growth.
- The halo must be removed by special absorbers called “collimators”.



Synchrotron radiation

- Any charged relativistic particle that is accelerated or decelerated emits radiation (cf Maxwell).
- When this is a transverse acceleration as in a dipole magnet this radiation is called synchrotron radiation.

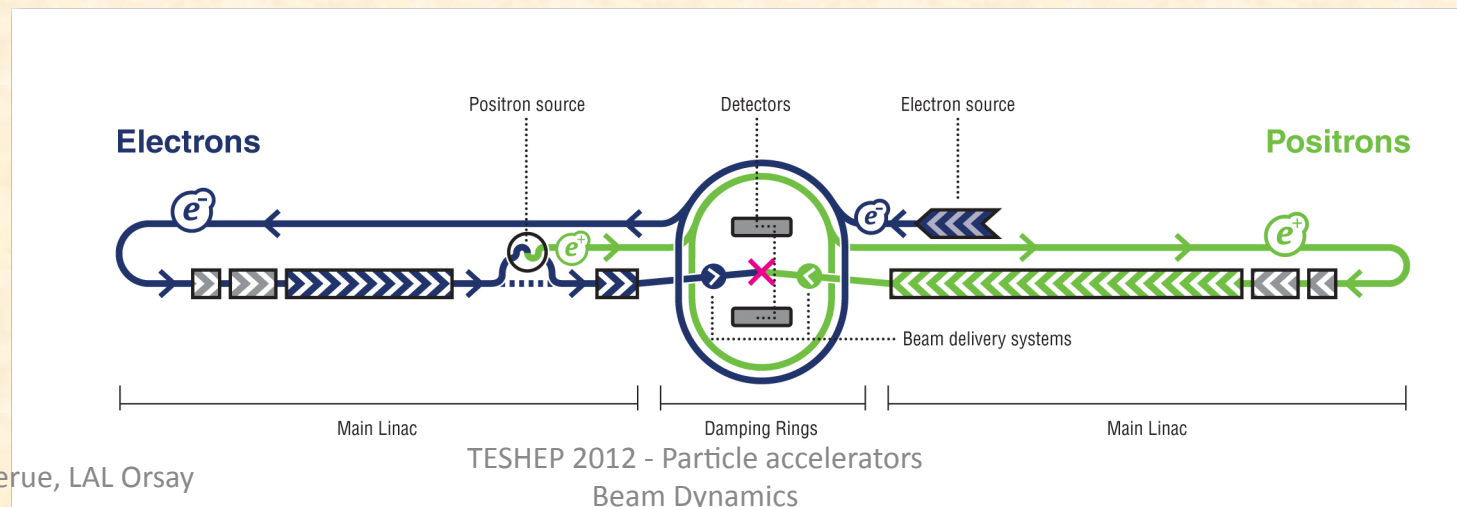


Discovery of Synchrotron radiation in 1946

Source: wikipedia

Radiation damping

- The energy lost due to synchrotron radiation emission has to be compensated by a RF cavity that tops-up the energy of the beam at each turn.
- This additional acceleration at each turn results in a decrease of the beam transverse emittance.
- By storing a beam in a ring for several milliseconds it is possible to significantly reduce its transverse emittance.
- The reduction of emittance in a ring due to SR emission is called “radiation damping”.
- As the radiation is emitted in the plane of the accelerator, radiation damping is faster in the direction orthogonal to the accelerator.
=> very flat beams



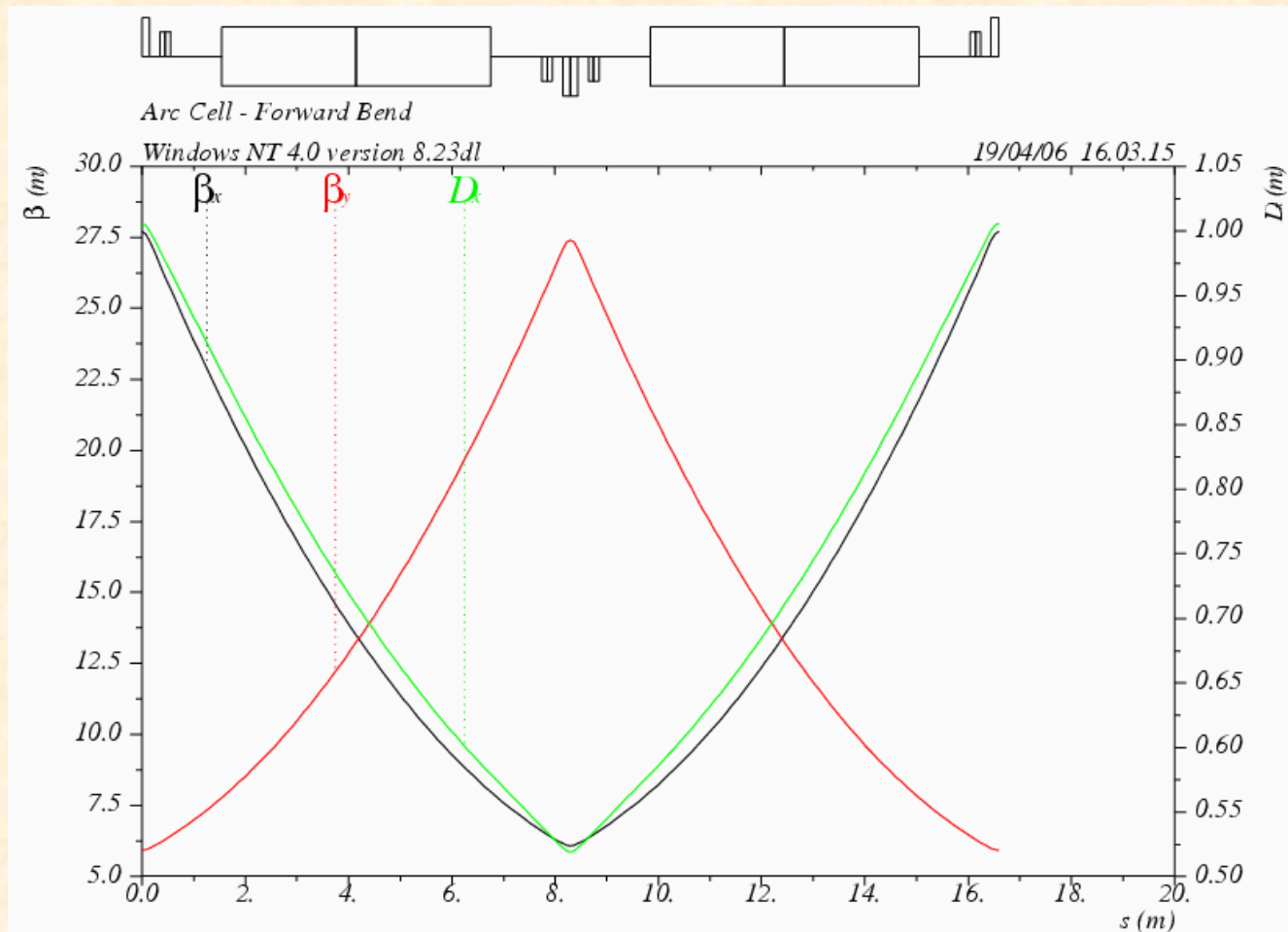
Lattice and emittance

- Radiation damping is limited by other phenomena such as emittance coupling and emittance growth.
- The minimum emittance that can be achieved depends on the lattice. It is given by

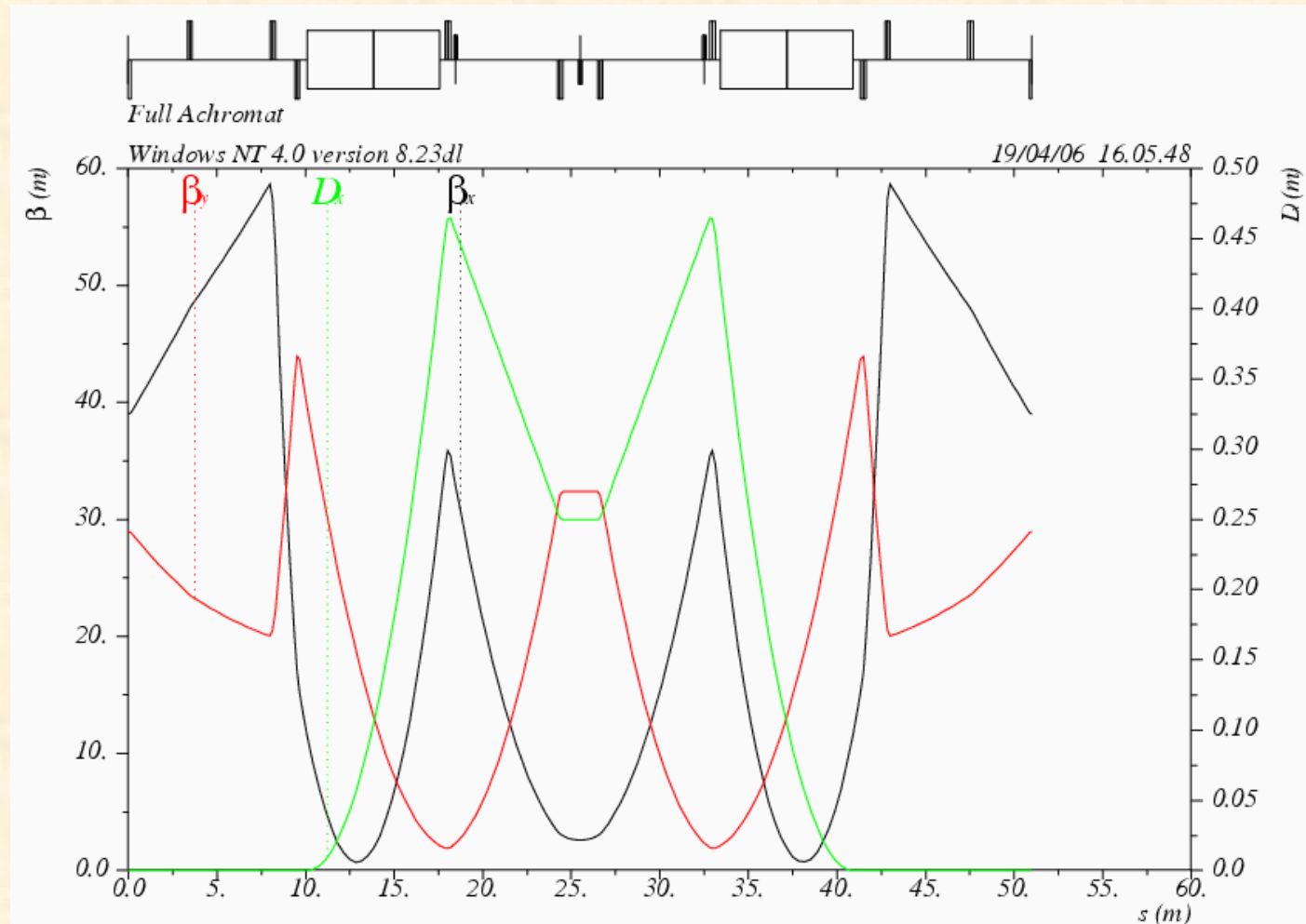
$$\varepsilon_{x,\min} = \frac{F}{12\sqrt{15}} C_q \gamma^2 \frac{\theta^3}{J_x}$$

- Where F is a factor depending on the lattice, theta the bending angle of each dipole, $C_q = 3.832 \times 10^{-19}$ m is a physical constant and J_x is the damping partition number.
- To achieve a low emittance (and therefore a high luminosity) the lattice must be carefully chosen.

FODO Lattice: $F \approx 100$

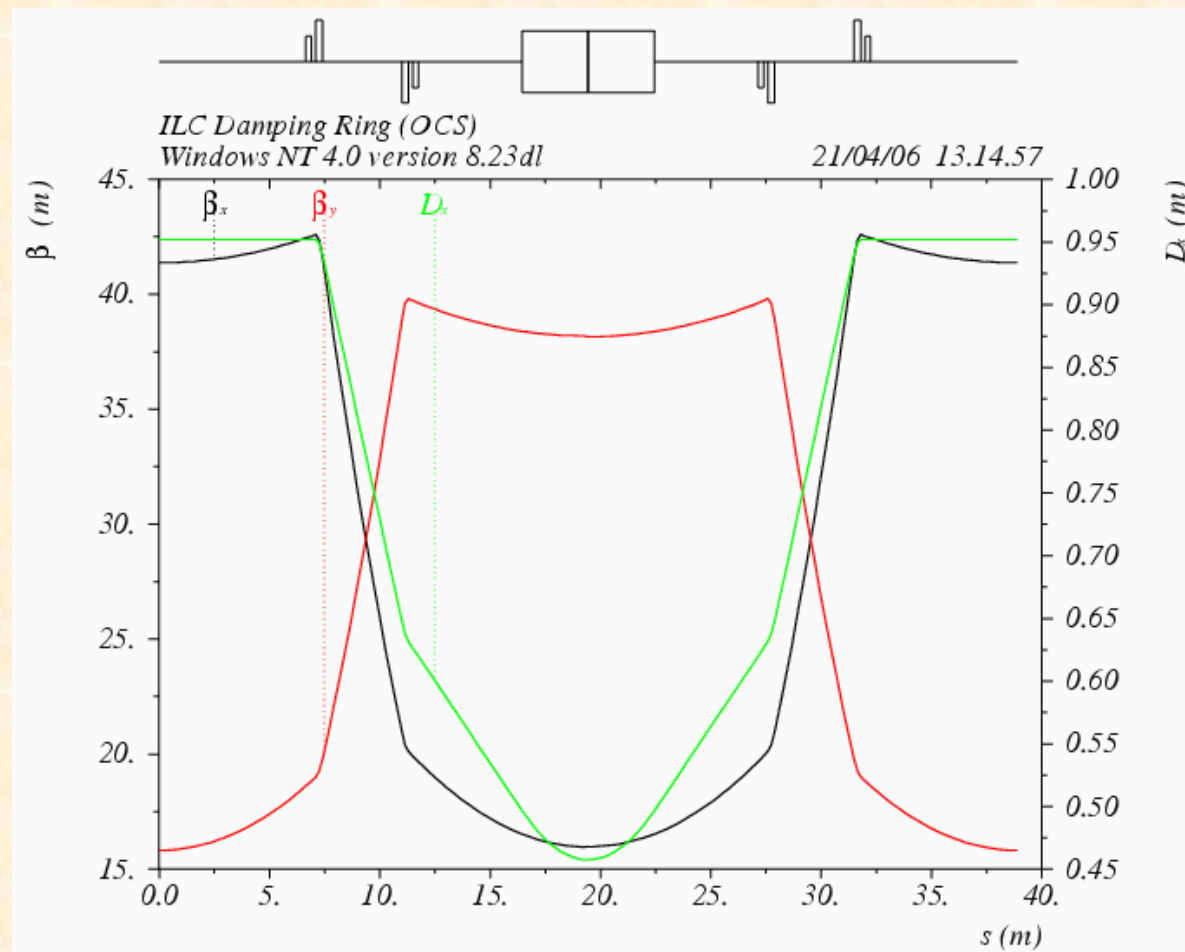


Double Bend Achromat (DBA) Lattice: $F = 3$



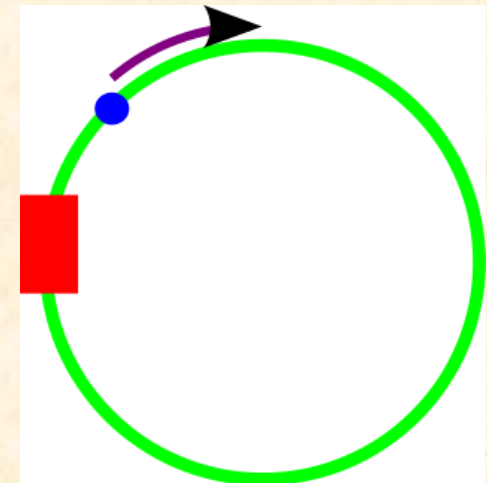
Theoretical Minimum Emittance (TME) Lattice:

$$F = 1$$



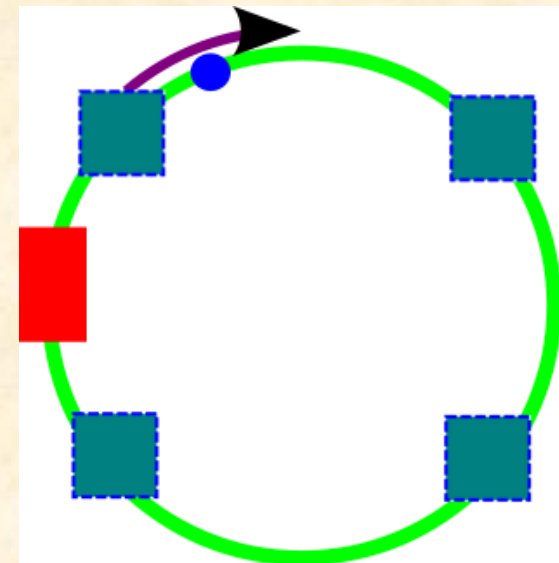
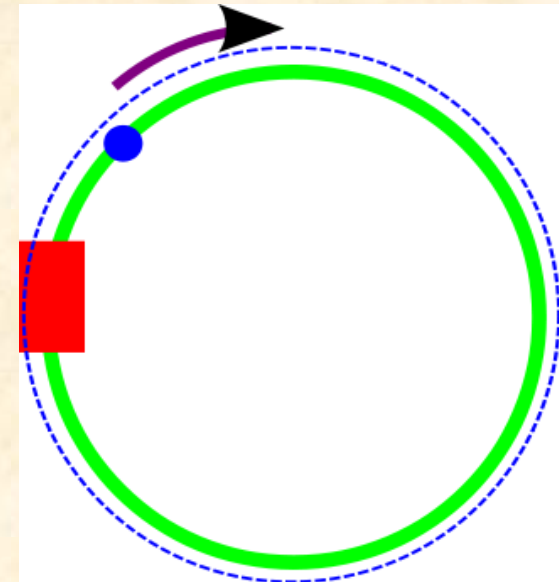
Quizz

- The particles in the beam do not all have the same energy. This is the “energy spread” of the beam.
- Let’s consider a ring with no focusing element (only dipoles).
- (1) How will the trajectory of the particle with more energy compare to that of particles with less energy?
 - (a) they will have a larger orbit
 - (b) they will have a narrower orbit
 - (c) their orbit will be similar
 - (d) can not tell



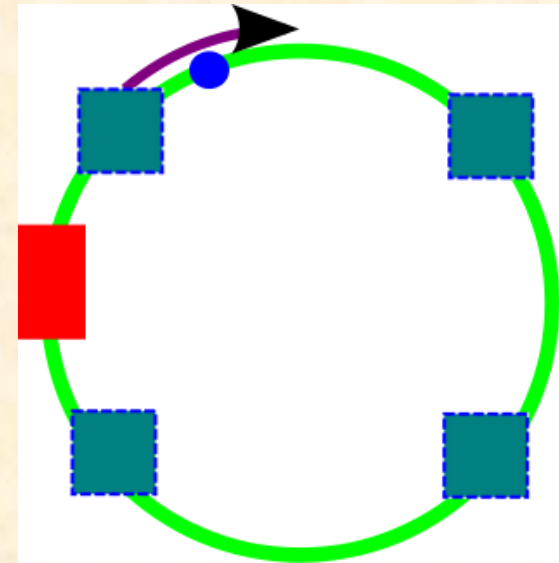
Answer (1a)

- Particles with a higher energy will be less deflected by the dipole magnets so they will have a larger orbit.
- (2) How is this result changed if there are 4 quadrupoles in the ring?
 - (a) they will have a larger orbit
 - (b) they will have a narrower orbit
 - (c) their orbit will be similar
 - (d) Can not tell



Answer (2d): Can not tell

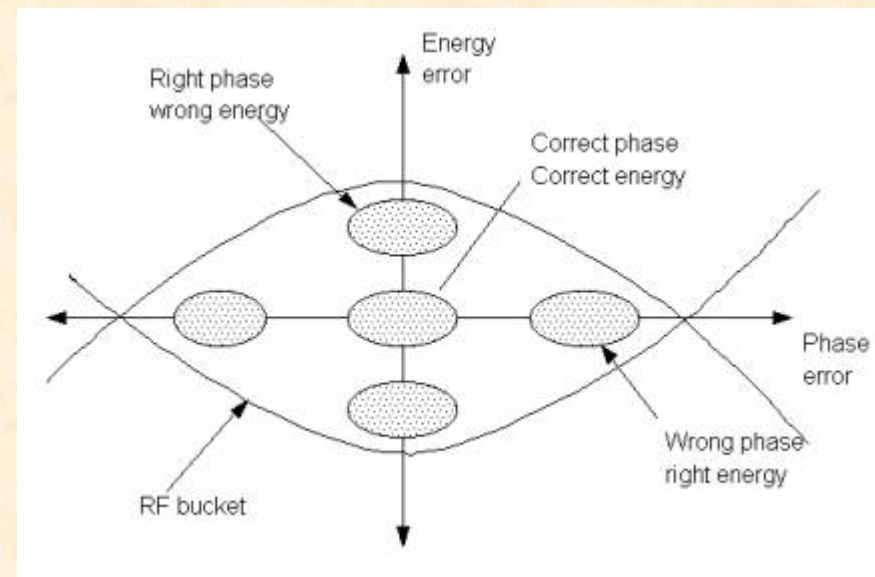
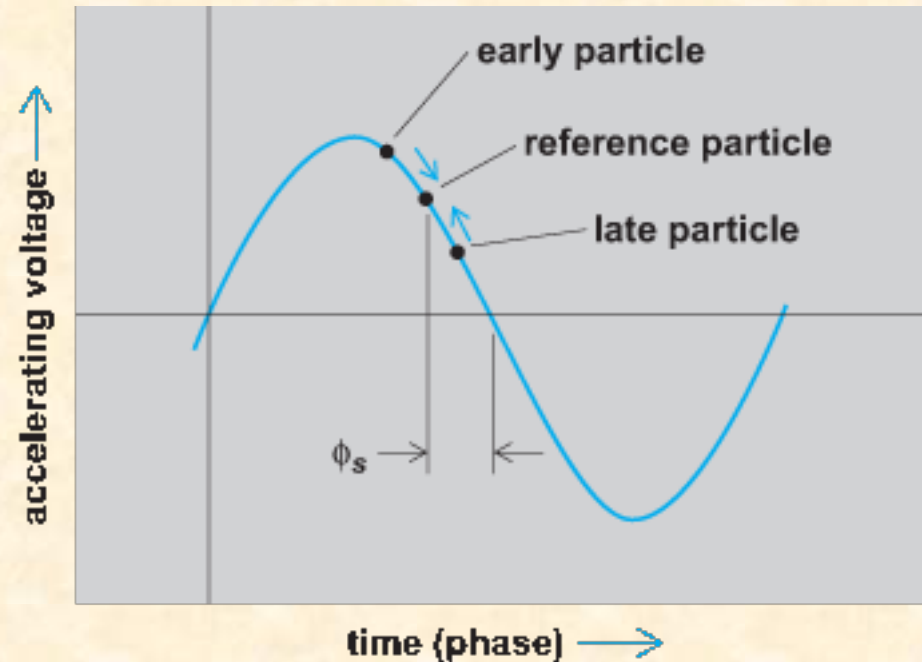
- To know which particles have a larger orbit and which particles have a narrower orbit the lattice must be taken into account.
- The momentum compaction factor gives the relation between momentum and orbit radius.
- It is dependant on the beam optics.



$$\frac{\Delta r}{r} = \alpha p \frac{\Delta p}{p}$$

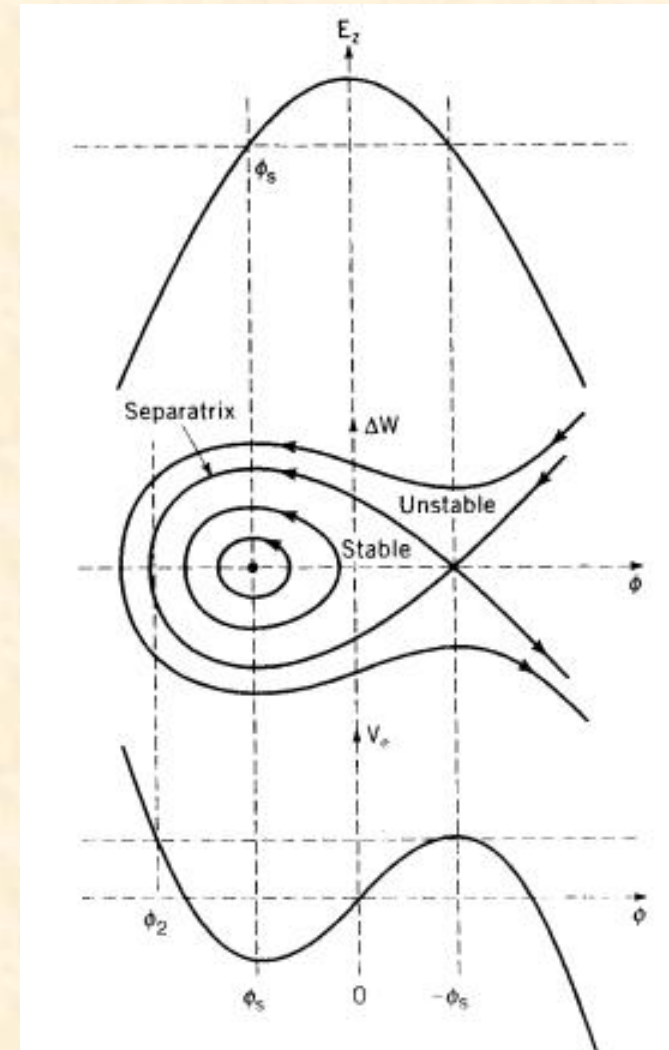
Longitudinal dynamics

- As there is a relation between momentum and orbit, the particles that do not have the correct momentum will experience a phase slippage.
- This will in turn induce a change in their energy.



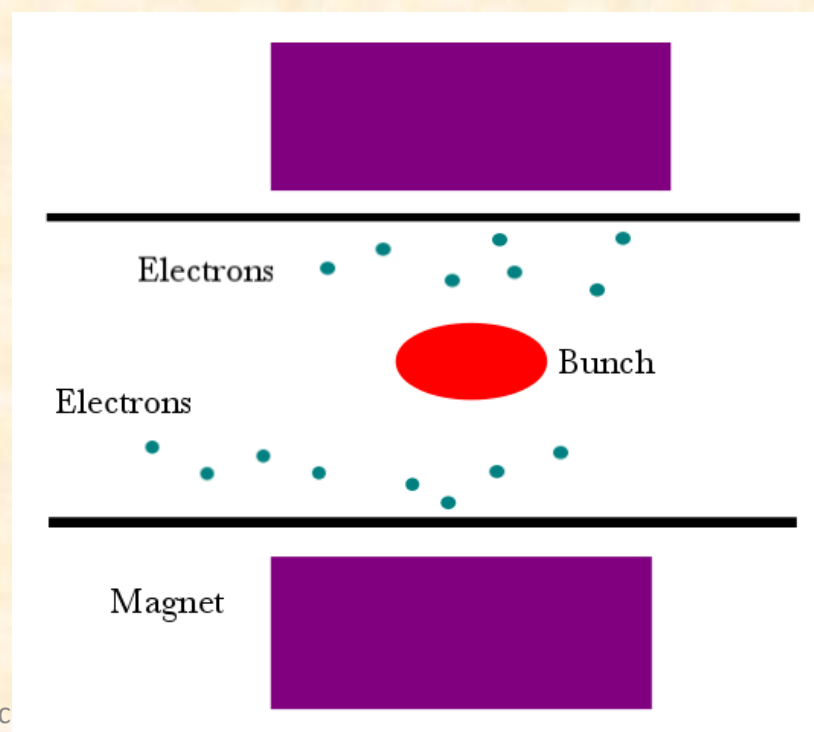
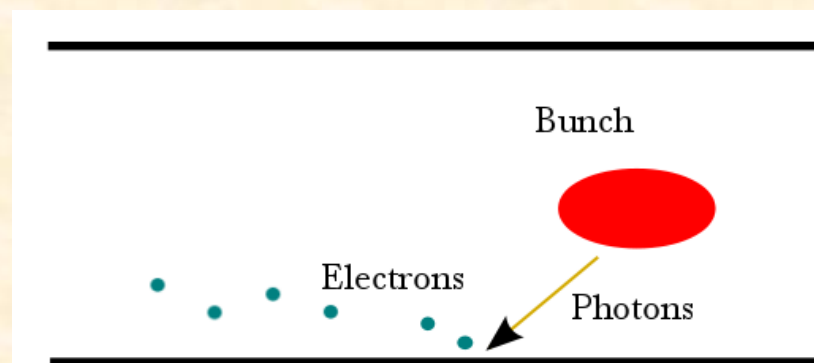
Longitudinal dynamics (2)

- Only a certain area of the phase vs energy plan is stable.
- Particles outside this stable area will drift away and eventually get lost.
- The position and size of this area will depend on the RF cavity voltage, the energy lost per turn (SR) and momentum compaction factor.



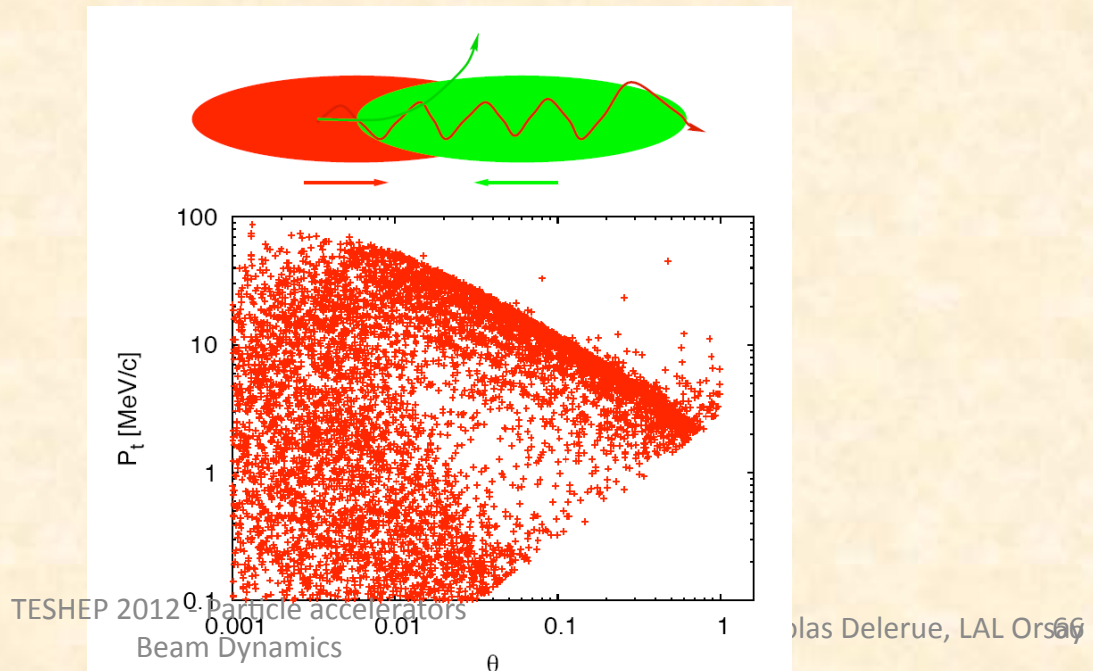
Electron cloud effect

- Radiation from a bunch can extract electrons (and ions) from the beam pipe and from residual gas in the vacuum.
- These electrons fall back and get re-absorbed with a certain time constant.
- However if the bunch frequency is too high these electrons (and ions) will accumulate in the beam pipe and shield the beam from the magnetic elements.
- Special coatings, beam pipe geometries and bunch repetition patterns can mitigate this problem to some extent.
- This is one of the main limitations to increasing the

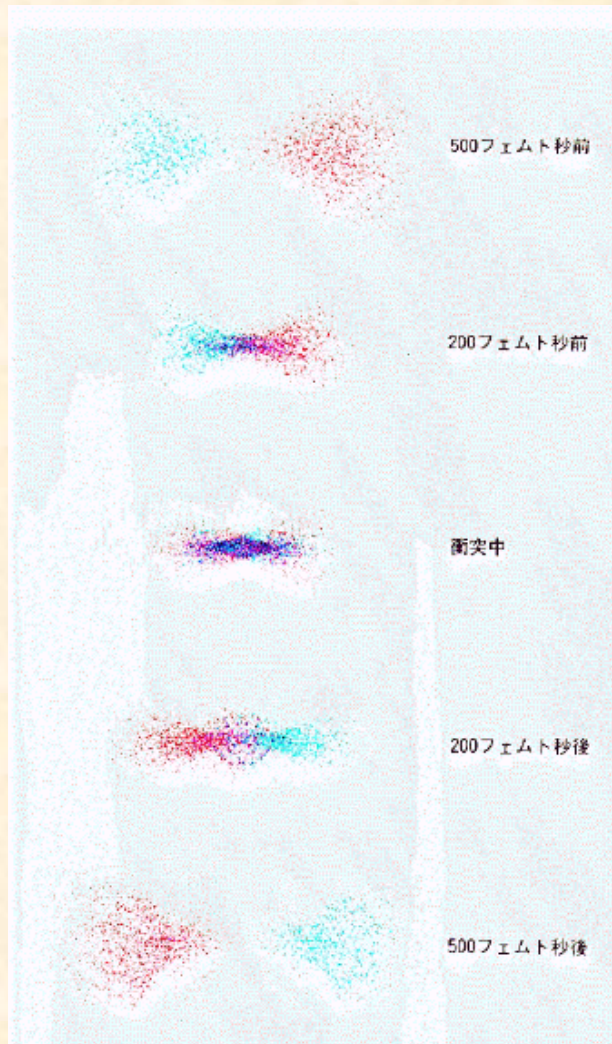


Beam beam effects (1)

- In a collider the two beams feel each other's electric field well before and well after colliding.
- Given that the particles come very close to each other, this lead to very intense forces.
- These forces lead to significant disruption of the beam.



Beam beam effect (2)



- At the interaction point the two beams self-focus onto each other.
- If the self focussing is too strong this can lead to a large emittance growth.
- If the two beam are not perfectly aligned this will also lead to large transverse deflection.
- This is a strong limitation on the size of the beams and therefore the luminosity, especially in a ring.

Summary

- Yesterday we learnt how to produce particles and anti-particles for colliders.
- Today we have seen how to control their trajectory (so that they can get injected in the LHC).
- We have also seen that the dynamics of the particles in the accelerator is very complicated.
- Great care must be taken to maintain the beam quality (emittance).
- Bunches too close from each other will damage each other by their wakefields or by electron cloud effect.
- Bunches compressed too much will blow up due to IBS, Touschek effect, space charge effect or beam-beam effect during the collisions.
- Tomorrow we will see how to check that the beam behaves as expected.

